CHAPTER 4
Trigonometric Functions

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CHAPTER 4
Trigonometric Functions

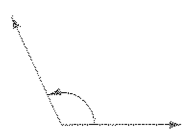
Section 4.1 Radian and Degree Measure

You should know the following basic facts about angles, their measurement, and their applications.

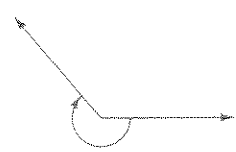
- Types of Angles:
  - (a) Acute: Measure between $0°$ and $90°$.
  - (b) Right: Measure $90°$.
  - (c) Obtuse: Measure between $90°$ and $180°$.
  - (d) Straight: Measure $180°$.
- α and β are complementary if $\alpha + \beta = 90°$. They are supplementary if $\alpha + \beta = 180°$.
- Two angles in standard position that have the same terminal side are called coterminal angles.
- To convert degrees to radians, use $1° = \pi/180$ radians.
- To convert radians to degrees, use 1 radian $= (180/\pi)°$.
- $1' =$ one minute $= 1/60$ of $1°$
- $1'' =$ one second $= 1/60$ of $1' = 1/3600$ of $1°$
- The length of a circular arc is $s = r\theta$ where θ is measured in radians.
- Speed = distance/time
- Angular speed $= \theta/t = s/rt$

Vocabulary Check

1. Trigonometry
2. angle
3. standard position
4. coterminal
5. radian
6. complementary
7. supplementary
8. degree
9. linear
10. angular

1. 

The angle shown is approximately 2 radians.

2. 

The angle shown is approximately -4 radians.

3. (a) Since $\frac{3\pi}{2} < \frac{7\pi}{4} < 2\pi$, $\frac{7\pi}{4}$ lies in Quadrant IV.

(b) Since $\frac{5\pi}{2} < \frac{11\pi}{4} < 3\pi$, $\frac{11\pi}{4}$ lies in Quadrant II.

4. (a) Since $-\frac{\pi}{2} < -\frac{5\pi}{12} < 0$, $\frac{5\pi}{2}$ lies in Quadrant IV.

(b) Since $-\frac{3\pi}{2} < -\frac{13\pi}{9} < -\pi$, $\frac{13\pi}{9}$ lies in Quadrant II.

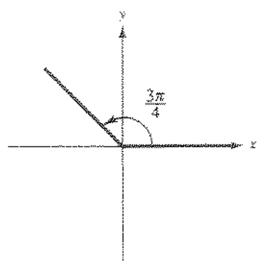
5. (a) Since $-\frac{\pi}{2} < -1 < 0$; -1 lies in Quadrant IV.

(b) Since $-\pi < -2 < -\frac{\pi}{2}$; -2 lies in Quadrant III.

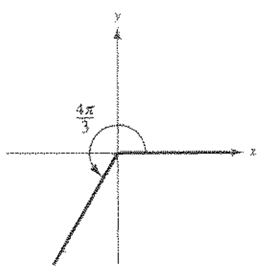
6. (a) Since $\pi < 3.5 < \frac{3\pi}{2}$, 3.5 lies in Quadrant III.

(b) Since $\frac{\pi}{2} < 2.25 < \pi$, 2.25 lies in Quadrant II.

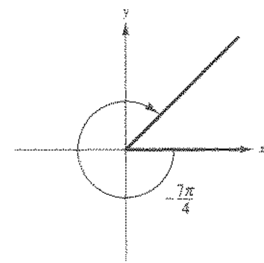
7. (a) $\frac{13\pi}{4}$



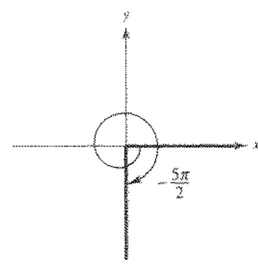
(b) $\frac{4\pi}{3}$



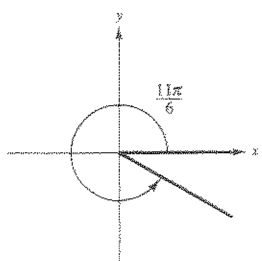
8. (a) $-\frac{7\pi}{4}$



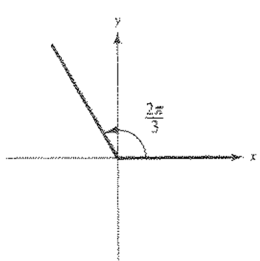
(b) $-\frac{5\pi}{2}$



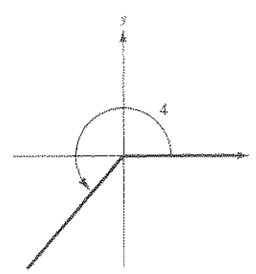
9. (a) $\frac{11\pi}{6}$



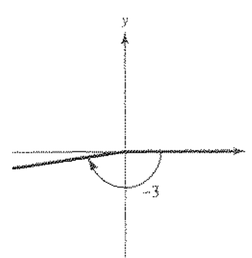
(b) $\frac{2\pi}{3}$



10. (a) 4



(b) -3



11. (a) Coterminal angles for $\frac{\pi}{6}$:

$$\frac{\pi}{6} + 2\pi = \frac{13\pi}{6}$$

$$\frac{\pi}{6} - 2\pi = -\frac{11\pi}{6}$$

(b) Coterminal angles for $\frac{2\pi}{3}$:

$$\frac{2\pi}{3} + 2\pi = \frac{8\pi}{3}$$

$$\frac{2\pi}{3} - 2\pi = -\frac{4\pi}{3}$$

12. (a) Coterminal angles for $\frac{7\pi}{6}$:

$$\frac{7\pi}{6} + 2\pi = \frac{19\pi}{6}$$
$$\frac{7\pi}{6} - 2\pi = -\frac{5\pi}{6}$$

(b) Coterminal angles for $\frac{5\pi}{4}$:

$$\frac{5\pi}{4} + 2\pi = \frac{13\pi}{4}$$
$$\frac{5\pi}{4} - 2\pi = -\frac{3\pi}{4}$$

13. (a) Coterminal angles for $-\frac{9\pi}{4}$:

$$-\frac{9\pi}{4} + 2\pi = -\frac{\pi}{4}$$
$$-\frac{9\pi}{4} + 4\pi = \frac{7\pi}{4}$$

(b) Coterminal angles for $-\frac{2\pi}{15}$:

$$-\frac{2\pi}{15} + 2\pi = \frac{28\pi}{15}$$
$$-\frac{2\pi}{15} - 2\pi = -\frac{32\pi}{15}$$

14. (a) Coterminal angles for $\frac{7\pi}{8}$:

$$\frac{7\pi}{8} + 2\pi = \frac{23\pi}{8}$$
$$\frac{7\pi}{8} - 2\pi = -\frac{9\pi}{8}$$

(b) Coterminal angles for $\frac{8\pi}{45}$:

$$\frac{8\pi}{45} + 2\pi = \frac{98\pi}{45}$$
$$\frac{8\pi}{45} - 2\pi = -\frac{82\pi}{45}$$

15. Complement: $\frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}$

Supplement: $\pi - \frac{\pi}{3} = \frac{2\pi}{3}$

16. Complement: Not possible; $\frac{3\pi}{4}$ is greater than $\frac{\pi}{2}$.

Supplement: $\pi - \frac{3\pi}{4} = \frac{\pi}{4}$

17. Complement: $\frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}$

Supplement: $\pi - \frac{\pi}{6} = \frac{5\pi}{6}$

18. Complement: Not possible; $\frac{2\pi}{3}$ is greater than $\frac{\pi}{2}$.

Supplement: $\pi - \frac{2\pi}{3} = \frac{\pi}{3}$

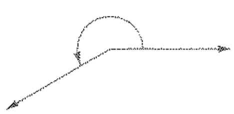
19. Complement: $\frac{\pi}{2} - 1 \approx 0.57$

Supplement: $\pi - 1 \approx 2.14$

20. Complement: None $\left(2 > \frac{\pi}{2}\right)$

Supplement: $\pi - 2 \approx 1.14$

21.



The angle shown is approximately 210°.

22.

The angle shown is approximately −45°.

23. (a) Since $90° < 150° < 180°$, 150° lies in Quadrant II.

(b) Since $270° < 282° < 360°$, 282° lies in Quadrant IV.

24. (a) Since $0° < 87.9° < 90°$, 87.9° lies in Quadrant I.

(b) Since $0° < 8.5° < 90°$, 8.5° lies in Quadrant I.

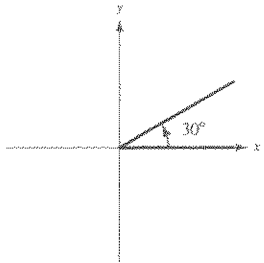
25. (a) Since $-180° < -132° 50' < -90°$, $-132° 50'$ lies in Quadrant III.

(b) Since $-360° < -336° 30' < -270°$, $-336° 30'$ lies in Quadrant I.

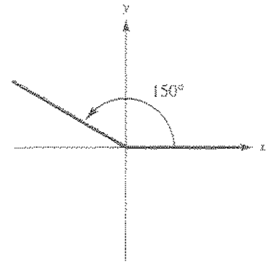
26. (a) Since $-270° < -245.25° < -180°$, $-245.25°$ lies in Quadrant II.

(b) Since $-90° < -12.35° < -0°$, $-12.35°$ lies in Quadrant IV.

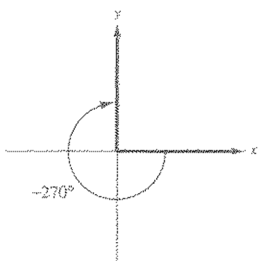
27. (a) 30°



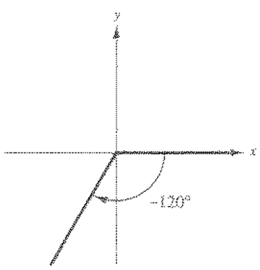
(b) 150°



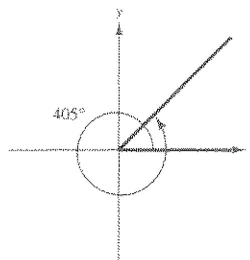
28. (a) -270°



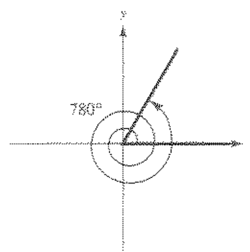
(b) -120°



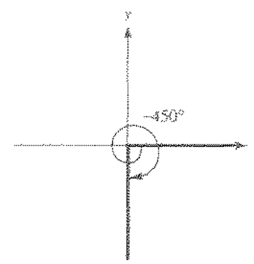
29. (a) 405°



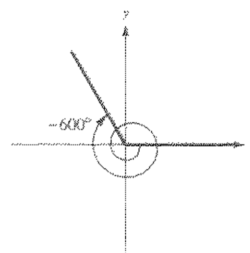
(b) 780°



30. (a) -450°



(b) -600°



31. (a) Coterminal angles for 52°:

$52^\circ + 360^\circ = 412^\circ$

$52^\circ - 360^\circ = -308^\circ$

(b) Coterminal angles for -36°:

$-36^\circ + 360^\circ = 324^\circ$

$-36^\circ - 360^\circ = -396^\circ$

32. (a) Coterminal angles for 114°:

$114^\circ + 360^\circ = 474^\circ$

$114^\circ - 360^\circ = -246^\circ$

(b) Coterminal angles for -390°:

$-390^\circ + 720^\circ = 330^\circ$

$-390^\circ + 360^\circ = -30^\circ$

33. (a) Coterminal angles for 300°:

$300^\circ + 360^\circ = 660^\circ$

$300^\circ - 360^\circ = -60^\circ$

(b) Coterminal angles for 230°:

$230^\circ + 360^\circ = 590^\circ$

$230^\circ - 360^\circ = -130^\circ$

34. (a) Coterminal angles for -445°:

$-445^\circ + 720^\circ = 275^\circ$

$-445^\circ + 360^\circ = -85^\circ$

(b) Coterminal angles for -740°:

$-740^\circ + 1080^\circ = 340^\circ$

$-740^\circ + 720^\circ = -20^\circ$

35. Complement: $90^\circ - 24^\circ = 66^\circ$

Supplement: $180^\circ - 24^\circ = 156^\circ$

36. Complement: Not possible

Supplement: $180^\circ - 129^\circ = 51^\circ$

37. Complement: $90^\circ - 87^\circ = 3^\circ$

Supplement: $180^\circ - 87^\circ = 93^\circ$

38. Complement: Not possible

Supplement: $180^\circ - 167^\circ = 13^\circ$

39. (a) $30^\circ = 30^\circ\left(\dfrac{\pi}{180^\circ}\right) = \dfrac{\pi}{6}$

(b) $150^\circ = 150^\circ\left(\dfrac{\pi}{180^\circ}\right) = \dfrac{5\pi}{6}$

40. (a) $315° = 315°\left(\frac{\pi}{180°}\right) = \frac{7\pi}{4}$

(b) $120° = 120°\left(\frac{\pi}{180°}\right) = \frac{2\pi}{3}$

41. (a) $-20° = -20°\left(\frac{\pi}{180°}\right) = -\frac{\pi}{9}$

(b) $-240° = -240°\left(\frac{\pi}{180°}\right) = -\frac{4\pi}{3}$

42. (a) $-270° = -270°\left(\frac{\pi}{180°}\right) = -\frac{3\pi}{2}$ (b) $144° = 144°\left(\frac{\pi}{180°}\right) = \frac{4\pi}{5}$

43. (a) $\frac{3\pi}{2} = \frac{3\pi}{2}\left(\frac{180°}{\pi}\right) = 270°$

(b) $-\frac{7\pi}{6} = -\frac{7\pi}{6}\left(\frac{180°}{\pi}\right) = -210°$

44. (a) $-4\pi = -4\pi\left(\frac{180°}{\pi}\right) = -720°$

(b) $3\pi = 3\pi\left(\frac{180°}{\pi}\right) = 540°$

45. (a) $\frac{7\pi}{3} = \frac{7\pi}{3}\left(\frac{180°}{\pi}\right) = 420°$

(b) $-\frac{13\pi}{60} = -\frac{13\pi}{60}\left(\frac{180°}{\pi}\right) = -39°$

46. (a) $-\frac{15\pi}{6} = -\frac{15\pi}{6}\left(\frac{180°}{\pi}\right) = -450°$

(b) $\frac{28\pi}{15} = \frac{28\pi}{15}\left(\frac{180°}{\pi}\right) = 336°$

47. $115° = 115\left(\frac{\pi}{180°}\right) \approx 2.007$ radians

48. $83.7° = 83.7°\left(\frac{\pi}{180°}\right) \approx 1.461$ radians

49. $-216.35° = -216.35\left(\frac{\pi}{180°}\right) \approx -3.776$ radians

50. $-46.52° = -46.52°\left(\frac{\pi}{180°}\right) \approx -0.812$ radians

51. $-0.78° = -0.78\left(\frac{\pi}{180°}\right) \approx -0.014$ radians

52. $395° = 395°\left(\frac{\pi}{180°}\right) \approx 6.894$ radians

53. $\frac{\pi}{7} = \frac{\pi}{7}\left(\frac{180°}{\pi}\right) \approx 25.714°$

54. $\frac{8\pi}{13} = \frac{8\pi}{13}\left(\frac{180°}{\pi}\right) \approx 110.769°$

55. $6.5\pi = 6.5\pi\left(\frac{180°}{\pi}\right) = 1170°$

56. $-4.2\pi = -4.2\pi\left(\frac{180°}{\pi}\right) = -756°$

57. $-2 = -2\left(\frac{180°}{\pi}\right) \approx -114.592°$

58. $-0.48 = -0.48\left(\frac{180°}{\pi}\right) \approx -27.502°$

59. $64°\,45' = 64° + \left(\frac{45}{60}\right)^{\circ} = 64.75°$

60. $-124°\,30' = -124.5°$

61. $85°\,18'\,30'' = 85° + \left(\frac{18}{60}\right)^{\circ} + \left(\frac{30}{3600}\right)^{\circ} \approx 85.308°$

62. $-408°\,16'\,25'' \approx -408.274°$

63. $-125°\,36'' = -125° - \left(\frac{36}{3600}\right)^{\circ} = -125.01°$

64. $330°\,25'' \approx 330.007°$

65. $280.6° = 280° + 0.6(60)' = 280°\,36'$

66. $-115.8° = -115°\,48'$

67. $-345.12° = -345°\,7'\,12''$

68. $310.75° = 310°\,45'$

69. $-0.355 = -0.355\left(\dfrac{180^\circ}{\pi}\right)$

$\approx -20.34^\circ = -20^\circ\, 20'\, 24''$

70. $0.7865 = 0.7865\left(\dfrac{180^\circ}{\pi}\right)$

$\approx 45.0631^\circ$

$= 45^\circ + (0.0631)(60')$

$= 45^\circ + 3' + 0.786(60'')$

$\approx 45^\circ\, 3'\, 47''$

71. $s = r\theta$

$6 = 5\theta$

$\theta = \frac{6}{5}$ radians

72. $s = r\theta$

$31 = 12\theta$

$\theta = \frac{31}{12} = 2\frac{7}{12}$ radians

73. $s = r\theta$

$32 = 7\theta$

$\theta = \frac{32}{7} = 4\frac{4}{7}$ radians

74. $s = r\theta$

$60 = 75\theta$

$\theta = \frac{60}{75} = \frac{4}{5}$ radians

Because the angle represented is clockwise, this angle is $-\frac{4}{5}$ radians.

75. The angles in radians are:

$0^\circ = 0$ $\qquad 135^\circ = \dfrac{3\pi}{4}$

$30^\circ = \dfrac{\pi}{6}$ $\qquad 180^\circ = \pi$

$45^\circ = \dfrac{\pi}{4}$ $\qquad 210^\circ = \dfrac{7\pi}{6}$

$60^\circ = \dfrac{\pi}{3}$ $\qquad 270^\circ = \dfrac{3\pi}{2}$

$90^\circ = \dfrac{\pi}{2}$ $\qquad 330^\circ = \dfrac{11\pi}{6}$

76. The angles in degrees are:

$\dfrac{\pi}{6} = 30^\circ$ $\qquad \dfrac{5\pi}{4} = 225^\circ$

$\dfrac{\pi}{4} = 45^\circ$ $\qquad \dfrac{4\pi}{3} = 240^\circ$

$\dfrac{\pi}{3} = 60^\circ$ $\qquad \dfrac{5\pi}{3} = 300^\circ$

$\dfrac{2\pi}{3} = 120^\circ$ $\qquad \dfrac{7\pi}{4} = 315^\circ$

$\dfrac{5\pi}{6} = 150^\circ$ $\qquad \pi = 180^\circ$

77. $s = r\theta$

$8 = 15\theta$

$\theta = \dfrac{8}{15}$ radians

78. $\theta = \dfrac{s}{r} = \dfrac{10}{22} = \dfrac{5}{11}$ radian

79. $s = r\theta$

$35 = 14.5\theta$

$\theta = \dfrac{70}{29} \approx 2.414$ radians

80. $r = 80$ kilometers, $s = 160$ kilometers

$\theta = \dfrac{s}{r} = \dfrac{160}{80} = 2$ radians

81. $s = r\theta$, θ in radians

$s = 14(180)\left(\dfrac{\pi}{180}\right) = 14\pi \approx 43.982$ inches

82. $r = 9$ feet, $\theta = 60^\circ = \dfrac{\pi}{3}$

$s = r\theta = 9\left(\dfrac{\pi}{3}\right) = 3\pi$ feet

83. $s = r\theta$, θ in radians

$s = 27\left(\dfrac{2\pi}{3}\right) = 18\pi$ meters ≈ 56.55 meters

84. $r = 12$ centimeters, $\theta = \dfrac{3\pi}{4}$

$s = r\theta = 12\left(\dfrac{3\pi}{4}\right) = 9\pi$ centimeters ≈ 28.27 cm

85. $r = \dfrac{s}{\theta} = \dfrac{36}{\pi/2} = \dfrac{72}{\pi}$ feet ≈ 22.92 feet

86. $r = \dfrac{s}{\theta} = \dfrac{3}{4\pi/3} = \dfrac{9}{4\pi}$ meters ≈ 0.72 meter

87. $r = \dfrac{s}{\theta} = \dfrac{82}{135°(\pi/180°)} = \dfrac{328}{3\pi}$ miles ≈ 34.80 miles

88. $r = \dfrac{s}{\theta} = \dfrac{8}{330°(\pi/180°)} = \dfrac{48}{11\pi}$ inches ≈ 1.39 inches

89. $\theta = 42°\,7'\,33'' - 25°\,46'\,32''$

$= 16°\,21'\,1'' \approx 0.28537$ radian

$s = r\theta = 4000(0.28537) \approx 1141.48$ miles

90. $r = 4000$ miles

$\theta = 31°\,46' + 26°\,8' = 57°\,54' \approx 1.0105$ rad

$s = r\theta = 4000(1.0105) \approx 4042.0$ miles

91. $\theta = \dfrac{s}{r} = \dfrac{450}{6378} \approx 0.07056$ radian $\approx 4.04°$

$\approx 4°\,2'\,33.02''$

92. $r = 6378$, $s = 400$

$\theta = \dfrac{s}{r} = \dfrac{400}{6378} \approx 0.0627$ rad $\approx 3.59°$

93. $\theta = \dfrac{s}{r} = \dfrac{2.5}{6} = \dfrac{25}{60} = \dfrac{5}{12}$ radian $\approx 23.87°$

94. $\theta = \dfrac{s}{r} = \dfrac{24}{5} = 4.8$ rad $\approx 275.02°$

95. (a) single axel: $1\frac{1}{2}$ revolutions $= 360° + 180° = 540°$

$= 2\pi + \pi = 3\pi$ radians

(b) double axel: $2\frac{1}{2}$ revolutions $= 720° + 180° = 900°$

$= 4\pi + \pi = 5\pi$ radians

(c) triple axel: $3\frac{1}{2}$ revolutions $= 1260° = 7\pi$ radians

96. Linear speed $= \dfrac{s}{t} = \dfrac{r\theta}{t} = \dfrac{(6400 + 1250)2\pi}{110} \approx 436.967$ km/min

97. (a) $\dfrac{\text{Revolutions}}{\text{Second}} = \dfrac{2400}{60} = 40$ rev/sec

Angular speed $= (2\pi)(40) = 80\pi$ rad/sec

(b) Radius of saw blade $= \dfrac{7.5}{2} = 3.75$ in.

Radius in feet $= \dfrac{3.75}{12} = 0.3125$ ft

Speed $= \dfrac{s}{t} = \dfrac{r\theta}{t} = r\dfrac{\theta}{t} = r(\text{angular speed})$

$= 0.3125(80\pi) = 78.54$ ft/sec

98. (a) $\dfrac{\text{Revolutions}}{\text{Second}} = \dfrac{4800}{60} = 80$ rev/sec

Angular speed $= (2\pi)(80) = 160\pi$ rad/sec

(b) Radius of saw blade $= \dfrac{7.25}{2} = 3.625$ in.

Radius in feet $= \dfrac{3.625}{12} \approx 0.3021$ ft

Speed $= \dfrac{s}{t} = \dfrac{r\theta}{t} = r\dfrac{\theta}{t} = r(\text{angular speed})$

$= 0.3021(160\pi) \approx 151.84$ ft/sec

99. (a) $\dfrac{\text{Revolutions}}{\text{Hour}} = \dfrac{480}{(1/60)} = 28{,}800 \text{ rev/hr}$

Angular speed $= 2\pi(28{,}800) = 57{,}600\pi$ rad/hr

Radius of wheel $= \dfrac{25/2}{(12 \text{ in./ft})(5280 \text{ ft/mi})} = \dfrac{5}{25{,}344}$ miles

Speed $= \dfrac{s}{t} = \dfrac{r\theta}{t} = r\dfrac{\theta}{t} = r(\text{angular speed})$

$= \dfrac{5}{25{,}344} \cdot 57{,}600\pi = \dfrac{125\pi}{11} \approx 35.70$ miles/hr

(b) Let $x =$ spin balance machine rate.

$70 = r(\text{angular speed}) = r\left(2\pi \cdot \dfrac{x}{(1/60)}\right) = \dfrac{5}{25{,}344}120\pi x \Longrightarrow x \approx 941.18$ rev/min

100. (a) $200 \le \dfrac{\text{revolutions}}{\text{minute}} \le 500$

$2\pi(200) \le \text{angular speed} \le 2\pi(500)$

$400\pi \text{ rad/min} \le \text{angular speed} \le 1000\pi \text{ rad/min}$

(b) Speed $= \dfrac{s}{t} = \dfrac{r\theta}{t} = r\dfrac{\theta}{t} = r(\text{angular speed}) = 6(\text{angular speed})$

$6(400\pi) \le \text{linear speed} \le 6(1000\pi)$

For the outermost track, 6000π cm/min

101. False, 1 radian $= \left(\dfrac{180}{\pi}\right)^\circ \approx 57.3^\circ$, so one radian is much larger than one degree.

102. No, -1260° is coterminal with 180°.

103. True: $\dfrac{2\pi}{3} + \dfrac{\pi}{4} + \dfrac{\pi}{12} = \dfrac{8\pi + 3\pi + \pi}{12} = \pi = 180^\circ$

104. (a) An angle is in standard position when the origin is the vertex and the initial side coincides with the positive x-axis.

(b) A negative angle is generated by a clockwise rotation.

(c) Angles that have the same initial and terminal sides are coterminal angles.

(d) An obtuse angle is between 90° and 180°.

105. If θ is constant, the length of the arc is proportional to the radius ($s = r\theta$), and hence increasing.

106. Let A be the area of a circular sector of radius r and central angle θ. Then

$$\frac{A}{\pi r^2} = \frac{\theta}{2\pi} \Longrightarrow A = \frac{1}{2}r^2\theta.$$

107. $A = \dfrac{1}{2}r^2\theta = \dfrac{1}{2}(10)^2 \cdot \dfrac{\pi}{3} = \dfrac{50}{3}\pi$ square meters

108. Because $s = r\theta$, $\theta = \dfrac{12}{15}$.

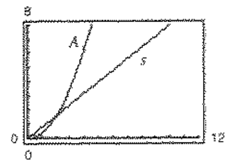
Hence, $A = \dfrac{1}{2}r^2\theta = \dfrac{1}{2}15^2\left(\dfrac{12}{15}\right) = 90 \text{ ft}^2$.

109. $A = \frac{1}{2}r^2\theta,\ s = r\theta$

(a) $\theta = 0.8 \implies A = \frac{1}{2}r^2(0.8) = 0.4r^2$ Domain: $r > 0$

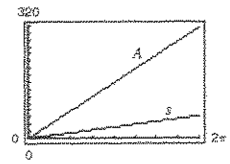
$s = r\theta = r(0.8)$ Domain: $r > 0$

The area function changes more rapidly for $r > 1$ because it is quadratic and the arc length function is linear.



(b) $r = 10 \implies A = \frac{1}{2}(10^2)\theta = 50\theta$ Domain: $0 < \theta < 2\pi$

$s = r\theta = 10\theta$ Domain: $0 < \theta < 2\pi$

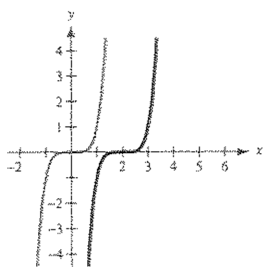


110. If a fan of greater diameter is installed, the angular speed does not change.

111. Answers will vary.

112. Answers will vary.

113.



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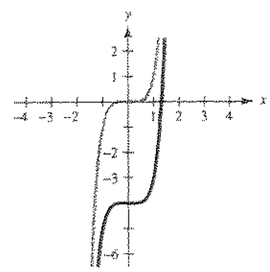

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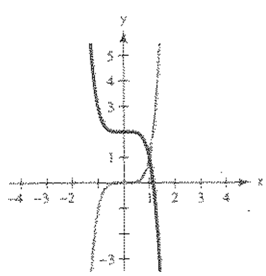

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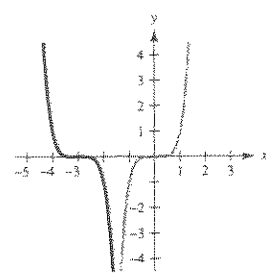

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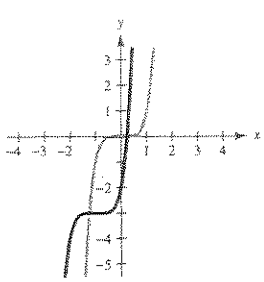

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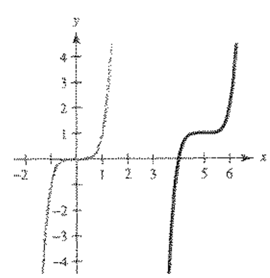


Section 4.2 Trigonometric Functions: The Unit Circle

- You should know how to evaluate trigonometric functions using the unit circle.
- You should know the definition of a *periodic* function.
- You should be able to recognize even and odd trigonometric functions.
- You should be able to evaluate trigonometric functions with a calculator in both radian and degree mode.

Vocabulary Check

1. unit circle 2. periodic 3. odd, even

1. $\sin\theta = y = \dfrac{15}{17}$

$\cos\theta = x = -\dfrac{8}{17}$

$\tan\theta = \dfrac{y}{x} = -\dfrac{15}{8}$

$\cot\theta = \dfrac{x}{y} = -\dfrac{8}{15}$

$\sec\theta = \dfrac{1}{x} = -\dfrac{17}{8}$

$\csc\theta = \dfrac{1}{y} = \dfrac{17}{15}$

2. $(x, y) = \left(\dfrac{12}{13}, \dfrac{5}{13}\right)$

$\sin\theta = y = \dfrac{5}{13}$

$\cos\theta = x = \dfrac{12}{13}$

$\tan\theta = \dfrac{y}{x} = \dfrac{5/13}{12/13} = \dfrac{5}{12}$

$\csc\theta = \dfrac{1}{y} = \dfrac{1}{5/13} = \dfrac{13}{5}$

$\sec\theta = \dfrac{1}{x} = \dfrac{1}{12/13} = \dfrac{13}{12}$

$\cot\theta = \dfrac{x}{y} = \dfrac{12/13}{5/13} = \dfrac{12}{5}$

3. $\sin\theta = y = -\dfrac{5}{13}$

$\cos\theta = x = \dfrac{12}{13}$

$\tan\theta = \dfrac{y}{x} = -\dfrac{5}{12}$

$\cot\theta = \dfrac{x}{y} = -\dfrac{12}{5}$

$\sec\theta = \dfrac{1}{x} = \dfrac{13}{12}$

$\csc\theta = \dfrac{1}{y} = -\dfrac{13}{5}$

4. $(x, y) = \left(-\dfrac{4}{5}, -\dfrac{3}{5}\right)$

$\sin\theta = y = -\dfrac{3}{5}$

$\cos\theta = x = -\dfrac{4}{5}$

$\tan\theta = \dfrac{y}{x} = \dfrac{-3/5}{-4/5} = \dfrac{3}{4}$

$\csc\theta = \dfrac{1}{y} = \dfrac{1}{-3/5} = -\dfrac{5}{3}$

$\sec\theta = \dfrac{1}{x} = \dfrac{1}{-4/5} = -\dfrac{5}{4}$

$\cot\theta = \dfrac{x}{y} = \dfrac{-4/5}{-3/5} = \dfrac{4}{3}$

5. $t = \dfrac{\pi}{4}$ corresponds to $\left(\dfrac{\sqrt{2}}{2}, \dfrac{\sqrt{2}}{2}\right)$.

6. $t = \dfrac{\pi}{3} \Longrightarrow \left(\dfrac{1}{2}, \dfrac{\sqrt{3}}{2}\right)$

7. $t = \frac{7\pi}{6}$ corresponds to $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$.

8. $t = \frac{5\pi}{4} \Longrightarrow \left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

9. $t = \frac{2\pi}{3}$ corresponds to $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$.

10. $t = \frac{5\pi}{3}$ corresponds to $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$.

11. $t = \frac{3\pi}{2}$ corresponds to $(0, -1)$.

12. $t = \pi \Longrightarrow (-1, 0)$

13. $t = -\frac{7\pi}{4}$ corresponds to $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$.

14. $t = -\frac{4\pi}{3}$ corresponds to $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$.

15. $t = -\frac{3\pi}{2}$ corresponds to $(0, 1)$.

16. $t = -2\pi$ corresponds to $(1, 0)$.

17. $t = \frac{\pi}{4}$ corresponds to $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$.

$\sin t = y = \frac{\sqrt{2}}{2}$

$\cos t = x = \frac{\sqrt{2}}{2}$

$\tan t = \frac{y}{x} = 1$

18. $t = \frac{\pi}{3}$ corresponds to $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$.

$\sin \frac{\pi}{3} = y = \frac{\sqrt{3}}{2}$

$\cos \frac{\pi}{3} = x = \frac{1}{2}$

$\tan \frac{\pi}{3} = \frac{y}{x} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$

19. $t = \frac{7\pi}{6}$ corresponds to $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$.

$\sin t = y = -\frac{1}{2}$

$\cos t = x = -\frac{\sqrt{3}}{2}$

$\tan t = \frac{y}{x} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

20. $t = -\frac{5\pi}{4}$ corresponds to $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$.

$\sin t = y = \frac{\sqrt{2}}{2}$

$\cos t = x = -\frac{\sqrt{2}}{2}$

$\tan t = \frac{y}{x} = -1$

21. $t = \frac{2\pi}{3}$ corresponds to $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$.

$\sin t = y = \frac{\sqrt{3}}{2}$

$\cos t = x = -\frac{1}{2}$

$\tan t = \frac{y}{x} = -\sqrt{3}$

22. $t = \frac{5\pi}{3}$ corresponds to $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$.

$\sin t = y = -\frac{\sqrt{3}}{2}$

$\cos t = x = \frac{1}{2}$

$\tan t = \frac{y}{x} = -\sqrt{3}$

23. $t = -\dfrac{5\pi}{3}$ corresponds to $\left(\dfrac{1}{2}, \dfrac{\sqrt{3}}{2}\right)$.

$\sin t = y = \dfrac{\sqrt{3}}{2}$

$\cos t = x = \dfrac{1}{2}$

$\tan t = \dfrac{y}{x} = \sqrt{3}$

24. $t = \dfrac{11\pi}{6}$ corresponds to $\left(\dfrac{\sqrt{3}}{2}, -\dfrac{1}{2}\right)$.

$\sin t = y = -\dfrac{1}{2}$

$\cos t = x = \dfrac{\sqrt{3}}{2}$

$\tan t = \dfrac{y}{x} = -\dfrac{\sqrt{3}}{3}$

25. $t = -\dfrac{\pi}{6}$ corresponds to $\left(\dfrac{\sqrt{3}}{2}, -\dfrac{1}{2}\right)$.

$\sin t = y = -\dfrac{1}{2}$

$\cos t = x = \dfrac{\sqrt{3}}{2}$

$\tan t = \dfrac{y}{x} = -\dfrac{\sqrt{3}}{3}$

26. $t = -\dfrac{3\pi}{4}$ corresponds to $\left(-\dfrac{\sqrt{2}}{2}, -\dfrac{\sqrt{2}}{2}\right)$.

$\sin\left(-\dfrac{3\pi}{4}\right) = y = -\dfrac{\sqrt{2}}{2}$

$\cos\left(-\dfrac{3\pi}{4}\right) = x = -\dfrac{\sqrt{2}}{2}$

$\tan\left(-\dfrac{3\pi}{4}\right) = \dfrac{y}{x} = 1$

27. $t = -\dfrac{7\pi}{4}$ corresponds to $\left(\dfrac{\sqrt{2}}{2}, \dfrac{\sqrt{2}}{2}\right)$.

$\sin t = y = \dfrac{\sqrt{2}}{2}$

$\cos t = x = \dfrac{\sqrt{2}}{2}$

$\tan t = \dfrac{y}{x} = 1$

28. $t = -\dfrac{4\pi}{3}$ corresponds to $\left(-\dfrac{1}{2}, \dfrac{\sqrt{3}}{2}\right)$.

$\sin t = y = \dfrac{\sqrt{3}}{2}$

$\cos t = x = -\dfrac{1}{2}$

$\tan t = \dfrac{y}{x} = -\sqrt{3}$

29. $t = -\dfrac{3\pi}{2}$ corresponds to $(0, 1)$.

$\sin t = y = 1$

$\cos t = x = 0$

$\tan t = \dfrac{y}{x}$ is undefined.

30. $t = -2\pi$ corresponds to $(1, 0)$.

$\sin(-2\pi) = y = 0$

$\cos(-2\pi) = x = 1$

$\tan(-2\pi) = \dfrac{y}{x} = \dfrac{0}{1} = 0$

31. $t = \dfrac{3\pi}{4}$ corresponds to $\left(-\dfrac{\sqrt{2}}{2}, \dfrac{\sqrt{2}}{2}\right)$.

$\sin t = y = \dfrac{\sqrt{2}}{2}$ $\qquad \csc t = \dfrac{1}{y} = \sqrt{2}$

$\cos t = x = -\dfrac{\sqrt{2}}{2}$ $\qquad \sec t = \dfrac{1}{x} = -\sqrt{2}$

$\tan t = \dfrac{y}{x} = -1$ $\qquad \cot t = \dfrac{x}{y} = -1$

32. $t = \frac{5\pi}{6}$ corresponds to $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$.

$\sin t = y = \frac{1}{2}$

$\cos t = x = -\frac{\sqrt{3}}{2}$

$\tan t = \frac{y}{x} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

$\csc t = \frac{1}{y} = 2$

$\sec t = \frac{1}{x} = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$

$\cot t = \frac{x}{y} = -\sqrt{3}$

33. $t = \frac{\pi}{2}$ corresponds to $(0, 1)$.

$\sin t = y = 1$ $\qquad$ $\csc t = \frac{1}{y} = 1$

$\cos t = x = 0$ $\qquad$ $\sec t = \frac{1}{x}$ is undefined.

$\tan t = \frac{y}{x}$ is undefined. $\qquad$ $\cot t = \frac{x}{y} = 0$

34. $t = \frac{3\pi}{2}$ corresponds to $(0, -1)$.

$\sin \frac{3\pi}{2} = y = -1$

$\cos \frac{3\pi}{2} = x = 0$

$\tan \frac{3\pi}{2} = \frac{y}{x} = \frac{-1}{0} \Longrightarrow$ undefined

$\csc \frac{3\pi}{2} = \frac{1}{y} = \frac{1}{-1} = -1$

$\sec \frac{3\pi}{2} = \frac{1}{x} = \frac{1}{0} \Longrightarrow$ undefined

$\cot \frac{3\pi}{2} = \frac{x}{y} = \frac{0}{-1} = 0$

35. $t = -\frac{2\pi}{3}$ corresponds to $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$.

$\sin t = y = -\frac{\sqrt{3}}{2}$ $\qquad$ $\csc t = -\frac{2\sqrt{3}}{3}$

$\cos t = x = -\frac{1}{2}$ $\qquad$ $\sec \theta = -2$

$\tan t = \frac{y}{x} = \sqrt{3}$ $\qquad$ $\cot \theta = \frac{\sqrt{3}}{3}$

36. $t = -\frac{7\pi}{4}$ corresponds to $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$.

$\sin\left(-\frac{7\pi}{4}\right) = y = \frac{\sqrt{2}}{2}$ $\qquad$ $\csc\left(-\frac{7\pi}{4}\right) = \frac{1}{y} = \sqrt{2}$

$\cos\left(-\frac{7\pi}{4}\right) = x = \frac{\sqrt{2}}{2}$ $\qquad$ $\sec\left(-\frac{7\pi}{4}\right) = \frac{1}{x} = \sqrt{2}$

$\tan\left(-\frac{7\pi}{4}\right) = \frac{y}{x} = 1$ $\qquad$ $\cot\left(-\frac{7\pi}{4}\right) = \frac{x}{y} = 1$

37. $\sin 5\pi = \sin \pi = 0$

38. Because $7\pi = 6\pi + \pi$:

$\cos 7\pi = \cos(6\pi + \pi) = \cos \pi = -1$

39. $\cos \frac{8\pi}{3} = \cos \frac{2\pi}{3} = -\frac{1}{2}$

40. Because $\frac{9\pi}{4} = 2\pi + \frac{\pi}{4}$:

$\sin \frac{9\pi}{4} = \sin\left(2\pi + \frac{\pi}{4}\right) = \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$

41. $\cos\left(-\frac{13\pi}{6}\right) = \cos\left(-\frac{\pi}{6}\right) = \cos\left(\frac{11\pi}{6}\right) = \frac{\sqrt{3}}{2}$

42. Because $-\frac{19\pi}{6} = -4\pi + \frac{5\pi}{6}$:

$\sin\left(-\frac{19\pi}{6}\right) = \sin\left(-4\pi + \frac{5\pi}{6}\right)$

$= \sin\left(\frac{5\pi}{6}\right) = \frac{1}{2}$

43. $\sin\left(-\frac{9\pi}{4}\right) = \sin\left(-\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

44. Because $-\frac{8\pi}{3} = -4\pi + \frac{4\pi}{3}$:

$\cos\left(-\frac{8\pi}{3}\right) = \cos\left(-4\pi + \frac{4\pi}{3}\right)$

$= \cos \frac{4\pi}{3} = -\frac{1}{2}$

45. $\sin t = \frac{1}{3}$

(a) $\sin(-t) = -\sin t = -\frac{1}{3}$

(b) $\csc(-t) = -\csc t = -3$

46. $\cos t = -\frac{3}{4}$

(a) $\cos(-t) = \cos t = -\frac{3}{4}$

(b) $\sec(-t) = \frac{1}{\cos(-t)} = \frac{1}{\cos t} = -\frac{4}{3}$

47. $\cos(-t) = -\frac{1}{5}$

(a) $\cos t = \cos(-t) = -\frac{1}{5}$

(b) $\sec(-t) = \frac{1}{\cos(-t)} = -5$

48. $\sin(-t) = \frac{3}{8}$

(a) $\sin t = -\sin(-t) = -\frac{3}{8}$

(b) $\csc t = \frac{1}{\sin(t)} = \frac{1}{-\sin(-t)} = -\frac{8}{3}$

49. $\sin t = \frac{4}{5}$

(a) $\sin(\pi - t) = \sin t = \frac{4}{5}$

(b) $\sin(t + \pi) = -\sin t = -\frac{4}{5}$

50. $\cos t = \frac{4}{5}$

(a) $\cos(\pi - t) = -\cos t = -\frac{4}{5}$

(b) $\cos(t + \pi) = -\cos t = -\frac{4}{5}$

51. $\sin \frac{7\pi}{9} \approx 0.6428$

52. $\tan \frac{2\pi}{5} \approx 3.0777$

53. $\cos \frac{11\pi}{5} \approx 0.8090$

54. $\sin \frac{11\pi}{9} \approx -0.6428$

55. $\csc 1.3 \approx 1.0378$

56. $\cot 3.7 = \frac{1}{\tan 3.7} \approx 1.6007$

57. $\cos(-1.7) \approx -0.1288$

58. $\cos(-2.5) \approx -0.8011$

59. $\csc 0.8 = \frac{1}{\sin 0.8} \approx 1.3940$

60. $\sec 1.8 = \frac{1}{\cos 1.8} \approx -4.4014$

61. $\sec 22.8 = \frac{1}{\cos 22.8} \approx -1.4486$

62. $\sin(-13.4) \approx -0.7404$

63. $\cot 2.5 = \frac{1}{\tan 2.5} \approx -1.3386$

64. $\tan 1.75 \approx -5.5204$

65. $\csc(-1.5) = \frac{1}{\sin(-1.5)} \approx -1.0025$

66. $\tan(-2.25) \approx 1.2386$

67. $\sec(-4.5) = \frac{1}{\cos(-4.5)} \approx -4.7439$

68. $\csc(-5.2) = \frac{1}{\sin(-5.2)} \approx 1.1319$

69. (a) $\sin 5 \approx -1$

(b) $\cos 2 \approx -0.4$

70. (a) $\sin 0.75 = y \approx 0.7$

(b) $\cos 2.5 = x \approx -0.8$

71. (a) $\sin t = 0.25$

$t \approx 0.25$ or 2.89

(b) $\cos t = -0.25$

$t \approx 1.82$ or 4.46

72. (a) $\sin t = -0.75$

$t \approx 4.0$ or $t \approx 5.4$

(b) $\cos t = 0.75$

$t \approx 0.72$ or $t \approx 5.56$

73.
$$\begin{aligned} I &= 5e^{-2t}\sin t \\ I(0.7) &= 5e^{-1.4}\sin 0.7 \\ &\approx 0.79 \text{ amperes} \end{aligned}$$

74. At $t = 1.4$,

$I \approx 5e^{-2(1.4)} \sin 1.4 \approx 0.30$ amperes.

75. $y(t) = \frac{1}{4}\cos 6t$

(a) $y(0) = \frac{1}{4}\cos 0 = 0.2500$ ft

(b) $y\left(\frac{1}{4}\right) = \frac{1}{4}\cos \frac{3}{2} \approx 0.0177$ ft

(c) $y\left(\frac{1}{2}\right) = \frac{1}{4}\cos 3 \approx -0.2475$ ft

76. $y(t) = \frac{1}{4}e^{-t}\cos 6t$

(a) $y(0) = \frac{1}{4}e^{-0}\cos(0) = \frac{1}{4}$ foot

(b)

t	0.50	1.02	1.54	2.07	2.59
y	−0.15	0.09	−0.05	0.03	−0.02

(c) The maximum displacements are decreasing because of friction, which is modeled by the e^{-t} term.

(d) $\frac{1}{4}e^{-t}\cos 6t = 0$

$$\begin{aligned} \cos 6t &= 0 \\ 6t &= \frac{\pi}{2}, \frac{3\pi}{2} \\ t &= \frac{\pi}{12}, \frac{\pi}{4} \end{aligned}$$

77. False. $\sin\left(\dfrac{-4\pi}{3}\right) = \dfrac{\sqrt{3}}{2} > 0$

78. True

79. False. 0 corresponds to $(1, 0)$.

80. True

81. (a) The points have y-axis symmetry.

(b) $\sin t_1 = \sin(\pi - t_1)$ since they have the same y-value.

(c) $-\cos t_1 = \cos(\pi - t_1)$ since the x-values have opposite signs.

82. (a) The points (x_1, y_1) and (x_2, y_2) are symmetric about the origin.

(b) Because of the symmetry of the points, you can make the conjecture that $\sin(t_1 + \pi) = -\sin t_1$.

(c) Because of the symmetry of the points, you can make the conjecture that $\cos(t_1 + \pi) = -\cos t_1$.

83. $\cos 1.5 \approx 0.0707$, $2 \cos 0.75 \approx 1.4634$

Thus, $\cos 2t \neq 2 \cos t$.

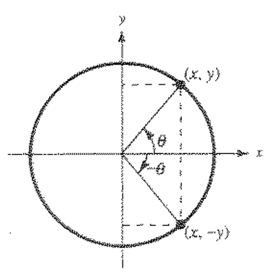
84. $\sin(0.25) + \sin(0.75) \approx 0.2474 + 0.6816 = 0.9290$

$$\sin 1 \approx 0.8415$$

Therefore, $\sin t_1 + \sin t_2 \neq \sin(t_1 + t_2)$.

85. $\cos\theta = x = \cos(-\theta)$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\cos(-\theta)} = \sec(-\theta)$$



86. $\sin\theta = y = -\sin(-\theta)$

$$\csc\theta = \frac{1}{y} = -\csc(-\theta)$$

$$\tan\theta = \frac{y}{x} = -\tan(-\theta)$$

$$\cot\theta = \frac{x}{y} = -\cot(-\theta)$$

87. $h(t) = f(t)g(t)$ is odd.

$h(-t) = f(-t)g(-t) = -f(t)g(t) = -h(t)$

88. $f(t) = \sin t$ and $g(t) = \tan t$

Both f and g are odd functions.

$h(t) = f(t)g(t) = \sin t \tan t$

$$\begin{aligned} h(-t) &= \sin(-t)\tan(-t) \\ &= (-\sin t)(-\tan t) \\ &= \sin t \tan t = h(t) \end{aligned}$$

The function $h(t) = f(t)g(t)$ is even.

89. $f(x) = \frac{1}{2}(3x - 2)$

$y = \frac{1}{2}(3x - 2)$

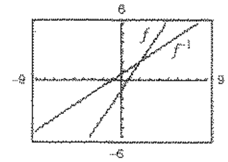
$x = \frac{1}{2}(3y - 2)$

$2x = 3y - 2$

$2x + 2 = 3y$

$\frac{2}{3}(x + 1) = y$

$f^{-1}(x) = \frac{2}{3}(x + 1)$



90. $f(x) = \frac{1}{4}x^3 + 1$

$y = \frac{1}{4}x^3 + 1$

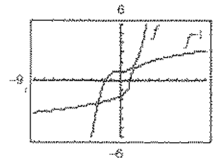
$x = \frac{1}{4}y^3 + 1$

$x - 1 = \frac{1}{4}y^3$

$4(x - 1) = y^3$

$y = \sqrt[3]{4(x - 1)}$

$f^{-1}(x) = \sqrt[3]{4(x - 1)}$



91. $f(x) = \sqrt{x^2 - 4},\ x \geq 2,\ y \geq 0$

$y = \sqrt{x^2 - 4}$

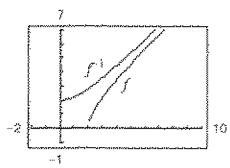
$x = \sqrt{y^2 - 4}$

$x^2 = y^2 - 4$

$x^2 + 4 = y^2$

$\sqrt{x^2 + 4} = y,\ x \geq 0$

$f^{-1}(x) = \sqrt{x^2 + 4},\ x \geq 0$



92. $f(x) = \dfrac{2x}{x + 1},\ x > -1$

$y = \dfrac{2x}{x + 1},\ x > -1$

$x = \dfrac{2y}{y + 1}$

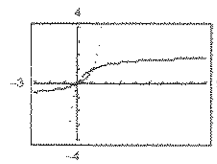
$xy + x = 2y$

$x = 2y - xy$

$x = y(2 - x)$

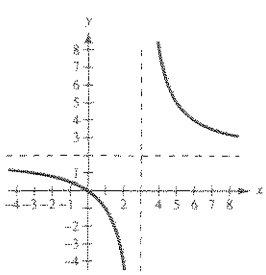
$\dfrac{x}{2 - x} = y,\ x < 2$

$f^{-1}(x) = \dfrac{x}{2 - x},\ x < 2$



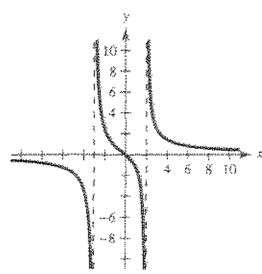
93. $f(x) = \dfrac{2x}{x - 3}$

Asymptotes: $x = 3,\ y = 2$



94. $f(x) = \dfrac{5x}{x^2 + x - 6} = \dfrac{5x}{(x + 3)(x - 2)}$

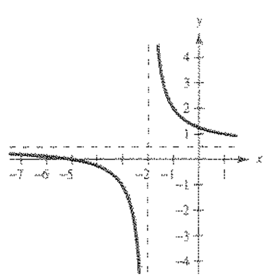
Asymptotes: $x = -3,\ x = 2,\ y = 0$



95. $f(x) = \dfrac{x^2 + 3x - 10}{2x^2 - 8} = \dfrac{(x - 2)(x + 5)}{2(x - 2)(x + 2)}$

$= \dfrac{x + 5}{2(x + 2)},\quad x \neq 2$

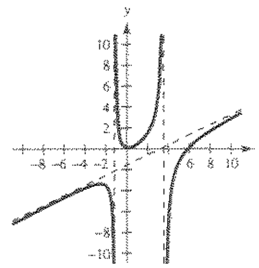
Asymptotes: $x = -2,\ y = \dfrac{1}{2}$



96. $f(x) = \dfrac{x^3 - 6x^2 + x - 1}{2x^2 - 5x - 8} = \dfrac{x}{2} - \dfrac{7}{4} - \dfrac{15(x + 4)}{4(2x^2 - 5x - 8)}$

Slant asymptote: $y = \dfrac{x}{2} - \dfrac{7}{4}$

Vertical asymptotes: $x \approx 3.608, x \approx -1.108$



97. $y = x^2 + 3x - 4$

Domain: all real numbers

$0 = x^2 + 3x - 4 = (x + 4)(x - 1) \Longrightarrow x = 1, -4$

Intercepts: $(0, -4), (1, 0), (-4, 0)$

No asymptotes

98. $y = \ln x^4$

Domain: all $x \neq 0$

$\ln x^4 = 0 \Longrightarrow x^4 = 1 \Longrightarrow x = \pm 1$

Intercepts: $(1, 0), (-1, 0)$

Asymptote: $x = 0$

99. $f(x) = 3^{x+1} + 2$

Domain: all real numbers

Intercept: $(0, 5)$

Asymptote: $y = 2$

100. $f(x) = \dfrac{x - 7}{(x^2 + 4x + 4)} = \dfrac{x - 7}{(x + 2)^2}$

Domain: all real numbers $x \neq -2$

Intercepts: $(7, 0), \left(0, -\dfrac{7}{4}\right)$

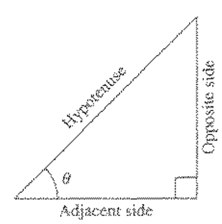
Asymptotes: $x = -2, y = 0$

Section 4.3 Right Triangle Trigonometry

- You should know the right triangle definition of trigonometric functions.

  (a) $\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$ (b) $\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$ (c) $\tan\theta = \dfrac{\text{opp}}{\text{adj}}$

  (d) $\csc\theta = \dfrac{\text{hyp}}{\text{opp}}$ (e) $\sec\theta = \dfrac{\text{hyp}}{\text{adj}}$ (f) $\cot\theta = \dfrac{\text{adj}}{\text{opp}}$



- You should know the following identities.

  (a) $\sin\theta = \dfrac{1}{\csc\theta}$ (b) $\csc\theta = \dfrac{1}{\sin\theta}$ (c) $\cos\theta = \dfrac{1}{\sec\theta}$

  (d) $\sec\theta = \dfrac{1}{\cos\theta}$ (e) $\tan\theta = \dfrac{1}{\cot\theta}$ (f) $\cot\theta = \dfrac{1}{\tan\theta}$

  (g) $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ (h) $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$ (i) $\sin^2\theta + \cos^2\theta = 1$

  (j) $1 + \tan^2\theta = \sec^2\theta$ (k) $1 + \cot^2\theta = \csc^2\theta$

- You should know that two acute angles α and β are complementary if $\alpha + \beta = 90°$, and cofunctions of complementary angles are equal.
- You should know the trigonometric function values of 30°, 45°, and 60°, or be able to construct triangles from which you can determine them.

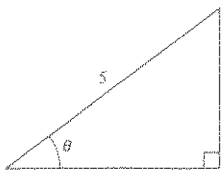
Vocabulary Check

1. (a) iii (b) vi (c) ii (d) v (e) i (f) iv

2. hypotenuse, opposite, adjacent

3. elevation, depression

1.



$\text{adj} = \sqrt{5^2 - 3^2} = \sqrt{16} = 4$

$\sin\theta = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{3}{5}$

$\cos\theta = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{4}{5}$

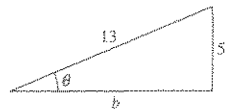
$\tan\theta = \dfrac{\text{opp}}{\text{adj}} = \dfrac{3}{4}$

$\csc\theta = \dfrac{\text{hyp}}{\text{opp}} = \dfrac{5}{3}$

$\sec\theta = \dfrac{\text{hyp}}{\text{adj}} = \dfrac{5}{4}$

$\cot\theta = \dfrac{\text{adj}}{\text{opp}} = \dfrac{4}{3}$

2.



$b = \sqrt{13^2 - 5^2} = \sqrt{169 - 25} = 12$

$\sin\theta = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{5}{13}$

$\cos\theta = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{12}{13}$

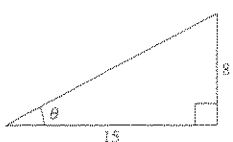
$\tan\theta = \dfrac{\text{opp}}{\text{adj}} = \dfrac{5}{12}$

$\csc\theta = \dfrac{\text{hyp}}{\text{opp}} = \dfrac{13}{5}$

$\sec\theta = \dfrac{\text{hyp}}{\text{adj}} = \dfrac{13}{12}$

$\cot\theta = \dfrac{\text{adj}}{\text{opp}} = \dfrac{12}{5}$

3.



$\text{hyp} = \sqrt{8^2 + 15^2} = 17$

$\sin\theta = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{8}{17}$

$\cos\theta = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{15}{17}$

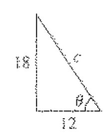
$\tan\theta = \dfrac{\text{opp}}{\text{adj}} = \dfrac{8}{15}$

$\csc\theta = \dfrac{\text{hyp}}{\text{opp}} = \dfrac{17}{8}$

$\sec\theta = \dfrac{\text{hyp}}{\text{adj}} = \dfrac{17}{15}$

$\cot\theta = \dfrac{\text{adj}}{\text{opp}} = \dfrac{15}{8}$

4.



$C = \sqrt{18^2 + 12^2} = \sqrt{468} = 6\sqrt{13}$

$\sin\theta = \dfrac{18}{6\sqrt{13}} = \dfrac{3\sqrt{13}}{13}$

$\cos\theta = \dfrac{12}{6\sqrt{13}} = \dfrac{2\sqrt{13}}{13}$

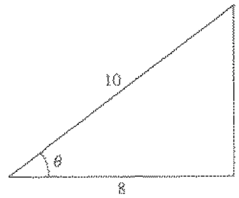
$\tan\theta = \dfrac{18}{12} = \dfrac{3}{2}$

$\csc\theta = \dfrac{\sqrt{13}}{3}$

$\sec\theta = \dfrac{\sqrt{13}}{2}$

$\cot\theta = \dfrac{2}{3}$

5. $\text{opp} = \sqrt{10^2 - 8^2} = 6$



$\sin\theta = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{6}{10} = \dfrac{3}{5}$

$\cos\theta = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{8}{10} = \dfrac{4}{5}$

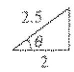
$\tan\theta = \dfrac{\text{opp}}{\text{adj}} = \dfrac{6}{8} = \dfrac{3}{4}$

$\csc\theta = \dfrac{\text{hyp}}{\text{opp}} = \dfrac{10}{6} = \dfrac{5}{3}$

$\sec\theta = \dfrac{\text{hyp}}{\text{adj}} = \dfrac{10}{8} = \dfrac{5}{4}$

$\cot\theta = \dfrac{\text{adj}}{\text{opp}} = \dfrac{8}{6} = \dfrac{4}{3}$

$\text{opp} = \sqrt{2.5^2 - 2^2} = 1.5$



$\sin\theta = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{1.5}{2.5} = \dfrac{3}{5}$

$\cos\theta = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{2}{2.5} = \dfrac{4}{5}$

$\tan\theta = \dfrac{\text{opp}}{\text{adj}} = \dfrac{1.5}{2} = \dfrac{3}{4}$

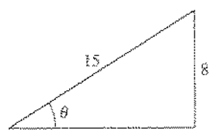
$\csc\theta = \dfrac{\text{hyp}}{\text{opp}} = \dfrac{2.5}{1.5} = \dfrac{5}{3}$

$\sec\theta = \dfrac{\text{hyp}}{\text{adj}} = \dfrac{2.5}{2} = \dfrac{5}{4}$

$\cot\theta = \dfrac{\text{adj}}{\text{opp}} = \dfrac{2}{1.5} = \dfrac{4}{3}$

The function values are the same since the triangles are similar and the corresponding sides are proportional.

6. $\text{adj} = \sqrt{15^2 - 8^2} = \sqrt{161}$



$\sin\theta = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{8}{15}$

$\cos\theta = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{\sqrt{161}}{15}$

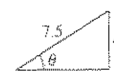
$\tan\theta = \dfrac{\text{opp}}{\text{adj}} = \dfrac{8}{\sqrt{161}} = \dfrac{8\sqrt{161}}{161}$

$\csc\theta = \dfrac{\text{hyp}}{\text{opp}} = \dfrac{15}{8}$

$\sec\theta = \dfrac{\text{hyp}}{\text{adj}} = \dfrac{15}{\sqrt{161}} = \dfrac{15\sqrt{161}}{161}$

$\cot\theta = \dfrac{\text{adj}}{\text{opp}} = \dfrac{\sqrt{161}}{8}$

$\text{adj} = \sqrt{7.5^2 - 4^2} = \dfrac{\sqrt{161}}{2}$



$\sin\theta = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{4}{7.5} = \dfrac{8}{15}$

$\cos\theta = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{\sqrt{161}}{2 \cdot 7.5} = \dfrac{\sqrt{161}}{15}$

$\tan\theta = \dfrac{\text{opp}}{\text{adj}} = \dfrac{4}{(\sqrt{161}/2)} = \dfrac{8}{\sqrt{161}} = \dfrac{8\sqrt{161}}{161}$

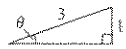
$\csc\theta = \dfrac{\text{hyp}}{\text{opp}} = \dfrac{7.5}{4} = \dfrac{15}{8}$

$\sec\theta = \dfrac{\text{hyp}}{\text{adj}} = \dfrac{7.5}{(\sqrt{161}/2)} = \dfrac{15}{\sqrt{161}} = \dfrac{15\sqrt{161}}{161}$

$\cot\theta = \dfrac{\text{adj}}{\text{opp}} = \dfrac{\sqrt{161}}{2 \cdot 4} = \dfrac{\sqrt{161}}{8}$

The function values are the same because the triangles are similar, and corresponding sides are proportional.

7. $\text{adj} = \sqrt{3^2 - 1^2} = \sqrt{8} = 2\sqrt{2}$



$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{1}{3}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{2\sqrt{2}}{3}$$

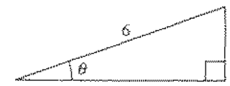
$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$$

$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = 3$$

$$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{3}{2\sqrt{2}} = \frac{3\sqrt{2}}{4}$$

$$\cot\theta = \frac{\text{adj}}{\text{opp}} = 2\sqrt{2}$$

$\text{adj} = \sqrt{6^2 - 2^2} = \sqrt{32} = 4\sqrt{2}$



$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{2}{6} = \frac{1}{3}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{4\sqrt{2}}{6} = \frac{2\sqrt{2}}{3}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{2}{4\sqrt{2}} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$$

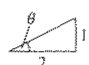
$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{6}{2} = 3$$

$$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{6}{4\sqrt{2}} = \frac{3}{2\sqrt{2}} = \frac{3\sqrt{2}}{4}$$

$$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$$

The function values are the same since the triangles are similar and the corresponding sides are proportional.

8. $\text{hyp} = \sqrt{1^2 + 2^2} = \sqrt{5}$



$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

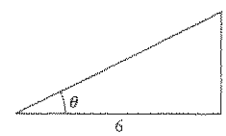
$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{1}{2}$$

$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{\sqrt{5}}{1} = \sqrt{5}$$

$$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{\sqrt{5}}{2}$$

$$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{2}{1} = 2$$

$\text{hyp} = \sqrt{3^2 + 6^2} = 3\sqrt{5}$



$$\sin\theta = \frac{3}{3\sqrt{5}} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$

$$\cos\theta = \frac{6}{3\sqrt{5}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

$$\tan\theta = \frac{3}{6} = \frac{1}{2}$$

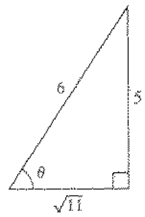
$$\csc\theta = \frac{3\sqrt{5}}{3} = \sqrt{5}$$

$$\sec\theta = \frac{3\sqrt{5}}{6} = \frac{\sqrt{5}}{2}$$

$$\cot\theta = \frac{6}{3} = 2$$

The function values are the same because the triangles are similar, and corresponding sides are proportional.

9. Given: $\sin\theta = \dfrac{5}{6} = \dfrac{\text{opp}}{\text{hyp}}$



$5^2 + (\text{adj})^2 = 6^2$

$\text{adj} = \sqrt{11}$

$\cos\theta = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{\sqrt{11}}{6}$

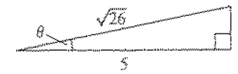
$\tan\theta = \dfrac{\text{opp}}{\text{adj}} = \dfrac{5}{\sqrt{11}} = \dfrac{5\sqrt{11}}{11}$

$\cot\theta = \dfrac{\text{adj}}{\text{opp}} = \dfrac{\sqrt{11}}{5}$

$\sec\theta = \dfrac{\text{hyp}}{\text{adj}} = \dfrac{6}{\sqrt{11}} = \dfrac{6\sqrt{11}}{11}$

$\csc\theta = \dfrac{\text{hyp}}{\text{opp}} = \dfrac{6}{5}$

10. $\text{hyp} = \sqrt{5^2 + 1^2} = \sqrt{26}$



$\sin\theta = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{1}{\sqrt{26}} = \dfrac{\sqrt{26}}{26}$

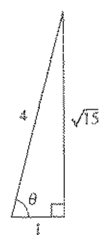
$\cos\theta = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{5}{\sqrt{26}} = \dfrac{5\sqrt{26}}{26}$

$\tan\theta = \dfrac{\text{opp}}{\text{adj}} = \dfrac{1}{5}$

$\csc\theta = \dfrac{\text{hyp}}{\text{opp}} = \dfrac{\sqrt{26}}{1} = \sqrt{26}$

$\sec\theta = \dfrac{\text{hyp}}{\text{adj}} = \dfrac{\sqrt{26}}{5}$

11. Given: $\sec\theta = 4 = \dfrac{4}{1} = \dfrac{\text{hyp}}{\text{adj}}$



$(\text{opp})^2 + 1^2 = 4^2$

$\text{opp} = \sqrt{15}$

$\sin\theta = \dfrac{\sqrt{15}}{4}$

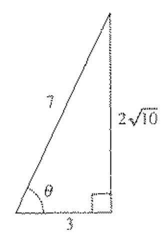
$\cos\theta = \dfrac{1}{4}$

$\tan\theta = \sqrt{15}$

$\cot\theta = \dfrac{1}{\sqrt{15}} = \dfrac{\sqrt{15}}{15}$

$\csc\theta = \dfrac{4}{\sqrt{15}} = \dfrac{4\sqrt{15}}{15}$

12. $\text{opp} = \sqrt{7^2 - 3^2} = \sqrt{40} = 2\sqrt{10}$



$\sin\theta = \dfrac{2\sqrt{10}}{7}$

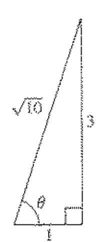
$\tan\theta = \dfrac{2\sqrt{10}}{3}$

$\csc\theta = \dfrac{7}{2\sqrt{10}} = \dfrac{7\sqrt{10}}{20}$

$\sec\theta = \dfrac{7}{3}$

$\cot\theta = \dfrac{3}{2\sqrt{10}} = \dfrac{3\sqrt{10}}{20}$

13. Given: $\tan\theta = 3 = \dfrac{3}{1} = \dfrac{\text{opp}}{\text{adj}}$



$3^2 + 1^2 = (\text{hyp})^2$

$\text{hyp} = \sqrt{10}$

$\sin\theta = \dfrac{3\sqrt{10}}{10}$

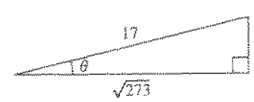
$\cos\theta = \dfrac{\sqrt{10}}{10}$

$\sec\theta = \sqrt{10}$

$\cot\theta = \dfrac{1}{3}$

$\csc\theta = \dfrac{\sqrt{10}}{3}$

14. $\text{adj} = \sqrt{17^2 - 4^2} = \sqrt{273}$



$\sin\theta = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{4}{17}$

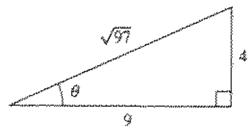
$\cos\theta = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{\sqrt{273}}{17}$

$\tan\theta = \dfrac{\text{opp}}{\text{adj}} = \dfrac{4}{\sqrt{273}} = \dfrac{4\sqrt{273}}{273}$

$\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{17}{\sqrt{273}} = \dfrac{17\sqrt{273}}{273}$

$\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{\sqrt{273}}{4}$

15. Given: $\cot\theta = \dfrac{9}{4} = \dfrac{\text{adj}}{\text{opp}}$



$4^2 + 9^2 = (\text{hyp})^2$

$\text{hyp} = \sqrt{97}$

$\sin\theta = \dfrac{4}{\sqrt{97}} = \dfrac{4\sqrt{97}}{97}$

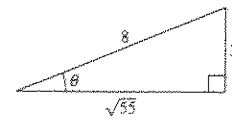
$\cos\theta = \dfrac{9}{\sqrt{97}} = \dfrac{9\sqrt{97}}{97}$

$\tan\theta = \dfrac{4}{9}$

$\sec\theta = \dfrac{\sqrt{97}}{9}$

$\csc\theta = \dfrac{\sqrt{97}}{4}$

16. $\sin\theta = \dfrac{3}{8}$



$\text{adj} = \sqrt{8^2 - 3^2} = \sqrt{55}$

$\cos\theta = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{\sqrt{55}}{8}$

$\tan\theta = \dfrac{\text{opp}}{\text{adj}} = \dfrac{3}{\sqrt{55}} = \dfrac{3\sqrt{55}}{55}$

$\csc\theta = \dfrac{1}{\sin\theta} = \dfrac{8}{3}$

$\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{8}{\sqrt{55}} = \dfrac{8\sqrt{55}}{55}$

$\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{\sqrt{55}}{3}$

	Function	*θ (deg)*	*θ (rad)*	*Function Value*
17.	sin	30°	$\frac{\pi}{6}$	$\frac{1}{2}$
18.	cos	45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$
19.	tan	60°	$\frac{\pi}{3}$	$\sqrt{3}$
20.	sec	45°	$\frac{\pi}{4}$	$\sqrt{2}$
21.	cot	60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{3}$
22.	csc	45°	$\frac{\pi}{4}$	$\sqrt{2}$
23.	cos	30°	$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$
24.	sin	45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$
25.	cot	45°	$\frac{\pi}{4}$	1
26.	tan	30°	$\frac{\pi}{6}$	$\frac{1}{\sqrt{3}}$

27. $\sin\theta = \dfrac{1}{\csc\theta}$ **28.** $\cos\theta = \dfrac{1}{\sec\theta}$ **29.** $\tan\theta = \dfrac{1}{\cot\theta}$ **30.** $\csc\theta = \dfrac{1}{\sin\theta}$

31. $\sec\theta = \dfrac{1}{\cos\theta}$ **32.** $\cot\theta = \dfrac{1}{\tan\theta}$ **33.** $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ **34.** $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

35. $\sin^2\theta + \cos^2\theta = 1$ **36.** $1 + \tan^2\theta = \sec^2\theta$ **37.** $\sin(90° - \theta) = \cos\theta$ **38.** $\cos(90° - \theta) = \sin\theta$

39. $\tan(90° - \theta) = \cot\theta$ **40.** $\cot(90° - \theta) = \tan\theta$ **41.** $\sec(90° - \theta) = \csc\theta$ **42.** $\csc(90° - \theta) = \sec\theta$

43. $\sin 60° = \dfrac{\sqrt{3}}{2}$, $\cos 60° = \dfrac{1}{2}$

(a) $\tan 60° = \dfrac{\sin 60°}{\cos 60°} = \sqrt{3}$

(b) $\sin 30° = \cos 60° = \dfrac{1}{2}$

(c) $\cos 30° = \sin 60° = \dfrac{\sqrt{3}}{2}$

(d) $\cot 60° = \dfrac{\cos 60°}{\sin 60°} = \dfrac{1}{\sqrt{3}} = \dfrac{\sqrt{3}}{3}$

44. $\sin 30° = \dfrac{1}{2}, \tan 30° = \dfrac{\sqrt{3}}{3}$

(a) $\csc 30° = \dfrac{1}{\sin 30°} = 2$

(b) $\cot 60° = \tan(90° - 60°) = \tan 30° = \dfrac{\sqrt{3}}{3}$

(c) $\cos 30° = \dfrac{\sin 30°}{\tan 30°} = \dfrac{(1/2)}{(\sqrt{3}/3)} = \dfrac{3}{2\sqrt{3}} = \dfrac{\sqrt{3}}{2}$

(d) $\cot 30° = \dfrac{1}{\tan 30°} = \dfrac{3}{\sqrt{3}} = \dfrac{3\sqrt{3}}{3} = \sqrt{3}$

45. $\csc \theta = 3, \ \sec \theta = \dfrac{3\sqrt{2}}{4}$

(a) $\sin \theta = \dfrac{1}{\csc \theta} = \dfrac{1}{3}$

(b) $\cos \theta = \dfrac{1}{\sec \theta} = \dfrac{2\sqrt{2}}{3}$

(c) $\tan \theta = \dfrac{\sin \theta}{\cos \theta} = \dfrac{1/3}{(2\sqrt{2})/3} = \dfrac{\sqrt{2}}{4}$

(d) $\sec(90° - \theta) = \csc \theta = 3$

46. $\sec \theta = 5, \tan \theta = 2\sqrt{6}$

(a) $\cos \theta = \dfrac{1}{\sec \theta} = \dfrac{1}{5}$

(b) $\cot \theta = \dfrac{1}{\tan \theta} = \dfrac{1}{2\sqrt{6}} = \dfrac{\sqrt{6}}{12}$

(c) $\cot(90° - \theta) = \tan \theta = 2\sqrt{6}$

(d) $\sin \theta = \tan \theta \cos \theta = (2\sqrt{6})\left(\dfrac{1}{5}\right) = \dfrac{2\sqrt{6}}{5}$

47. $\cos \alpha = \dfrac{1}{4}$

(a) $\sec \alpha = \dfrac{1}{\cos \alpha} = 4$

(b) $\sin^2 \alpha + \cos^2 \alpha = 1$

$$\sin^2 \alpha + \left(\frac{1}{4}\right)^2 = 1$$

$$\sin^2 \alpha = \frac{15}{16}$$

$$\sin \alpha = \frac{\sqrt{15}}{4}$$

(c) $\cot \alpha = \dfrac{\cos \alpha}{\sin \alpha}$

$$= \frac{1/4}{\sqrt{15}/4}$$

$$= \frac{1}{\sqrt{15}} = \frac{\sqrt{15}}{15}$$

(d) $\sin(90° - \alpha) = \cos \alpha = \dfrac{1}{4}$

48. $\tan \beta = 5$ (β lies in Quadrant I or III.)

(a) $\cot \beta = \dfrac{1}{\tan \beta} = \dfrac{1}{5}$

(b) $\sec^2 \beta = 1 + \tan^2 \beta \Longrightarrow \cos \beta = \dfrac{1}{\sqrt{1 + \tan^2 \beta}} = \dfrac{1}{\sqrt{1 + 25}} = \dfrac{1}{\sqrt{26}} = \dfrac{\sqrt{26}}{26}$

(c) $\tan(90° - \beta) = \cot \beta = \dfrac{1}{5}$

(d) $\csc \beta = \sqrt{1 + \cot^2 \beta} = \sqrt{1 + \dfrac{1}{25}} = \dfrac{\sqrt{26}}{5}$

49. $\tan \theta \cot \theta = \tan \theta\left(\dfrac{1}{\tan \theta}\right) = 1$

50. $\csc \theta \tan \theta = \dfrac{1}{\sin \theta} \cdot \dfrac{\sin \theta}{\cos \theta} = \dfrac{1}{\cos \theta} = \sec \theta$

51. $\tan \theta \cos \theta = \left(\dfrac{\sin \theta}{\cos \theta}\right) \cos \theta = \sin \theta$

52. $\cot \theta \sin \theta = \dfrac{\cos \theta}{\sin \theta} \sin \theta = \cos \theta$

53. $(1 + \cos\theta)(1 - \cos\theta) = 1 - \cos^2\theta$

$= (\sin^2\theta + \cos^2\theta) - \cos^2\theta$

$= \sin^2\theta$

54. $(\csc\theta + \cot\theta)(\csc\theta - \cot\theta) = \csc^2\theta - \cot^2\theta$

$= 1$

55. $\dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta} = \dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$

$= \dfrac{1}{\sin\theta\cos\theta}$

$= \dfrac{1}{\sin\theta} \cdot \dfrac{1}{\cos\theta} = \csc\theta\sec\theta$

56. $\dfrac{\tan\theta + \cot\theta}{\tan\theta} = \dfrac{\tan\theta}{\tan\theta} + \dfrac{\cot\theta}{\tan\theta}$

$= 1 + \dfrac{\cot\theta}{(1/\cot\theta)}$

$= 1 + \cot^2\theta = \csc^2\theta$

57. (a) $\sin 41° \approx 0.6561$

(b) $\cos 87° \approx 0.0523$

58. (a) $\tan 18.5° \approx 0.3346$

(b) $\cot 71.5° = \dfrac{1}{\tan 71.5°} \approx 0.3346$

59. (a) $\sec 42°\,12' = \sec 42.2° = \dfrac{1}{\cos 42.2°} \approx 1.3499$

(b) $\csc 48°\,7' = \dfrac{1}{\sin\left(48 + \frac{7}{60}\right)°} \approx 1.3432$

60. (a) $\cos(8°\,50'\,25'') = \cos\left(8 + \dfrac{50}{60} + \dfrac{25}{3600}\right)$

$\approx \cos(8.840278) \approx 0.9881$

(b) $\sec(8°\,50'\,25'') = \dfrac{1}{\cos(8°\,50'\,25'')} \approx 1.0120$

61. Make sure that your calculator is in radian mode.

(a) $\cot\dfrac{\pi}{16} = \dfrac{1}{\tan(\pi/16)} \approx 5.0273$

(b) $\tan\dfrac{\pi}{8} \approx 0.4142$

62. (a) $\sec(1.54) = \dfrac{1}{\cos(1.54)} \approx 32.4765$

(b) $\cos(1.25) \approx 0.3153$

63. (a) $\sin\theta = \dfrac{1}{2} \implies \theta = 30° = \dfrac{\pi}{6}$

(b) $\csc\theta = 2 \implies \theta = 30° = \dfrac{\pi}{6}$

64. (a) $\cos\theta = \dfrac{\sqrt{2}}{2} \implies \theta = 45° = \dfrac{\pi}{4}$

(b) $\tan\theta = 1 \implies \theta = 45° = \dfrac{\pi}{4}$

65. (a) $\sec\theta = 2 \implies \theta = 60° = \dfrac{\pi}{3}$

(b) $\cot\theta = 1 \implies \theta = 45° = \dfrac{\pi}{4}$

66. (a) $\tan\theta = \sqrt{3} \implies \theta = 60° = \dfrac{\pi}{3}$

(b) $\cos\theta = \dfrac{1}{2} \implies \theta = 60° = \dfrac{\pi}{3}$

67. (a) $\csc\theta = \dfrac{2\sqrt{3}}{3} \implies \theta = 60° = \dfrac{\pi}{3}$

(b) $\sin\theta = \dfrac{\sqrt{2}}{2} \implies \theta = 45° = \dfrac{\pi}{4}$

68. (a) $\cot\theta = \dfrac{\sqrt{3}}{3}$

$\tan\theta = \dfrac{3}{\sqrt{3}} = \sqrt{3} \implies \theta = 60° = \dfrac{\pi}{3}$

(b) $\sec\theta = \sqrt{2}$

$\cos\theta = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2} \implies \theta = 45° = \dfrac{\pi}{4}$

69. $\tan 30° = \dfrac{y}{105} \implies y = 105 \tan 30° = 105 \cdot \dfrac{\sqrt{3}}{3} = 35\sqrt{3}$

$\cos 30° = \dfrac{105}{r} \implies r = \dfrac{105}{\cos 30°} = \dfrac{105}{\sqrt{3}/2} = \dfrac{210}{\sqrt{3}} = 70\sqrt{3}$

70. $\cos 30° = \dfrac{x}{15} \implies x = 15 \cos 30° = 15 \cdot \dfrac{\sqrt{3}}{2} = \dfrac{15\sqrt{3}}{2}$

$\sin 30° = \dfrac{y}{15} \implies y = 15 \sin 30° = 15\left(\dfrac{1}{2}\right) = \dfrac{15}{2}$

71. $\cos 60° = \dfrac{x}{16} \implies x = 16 \cos 60° = 16\left(\dfrac{1}{2}\right) = 8$

$\sin 60° = \dfrac{y}{16} \implies y = 16 \sin 60° = 16\left(\dfrac{\sqrt{3}}{2}\right) = 8\sqrt{3}$

72. $\cot 60° = \dfrac{x}{38} \implies x = 38 \cot 60° = 38 \cdot \dfrac{1}{\sqrt{3}} = \dfrac{38\sqrt{3}}{3}$

$\sin 60° = \dfrac{38}{r} \implies r = \dfrac{38}{\sin 60°} = \dfrac{38}{\sqrt{3}/2} = \dfrac{76\sqrt{3}}{3}$

73. $\tan 45° = \dfrac{20}{x} \implies 1 = \dfrac{20}{x} \implies x = 20$

$r^2 = 20^2 + 20^2 \implies r = 20\sqrt{2}$

74. $\tan 45° = \dfrac{y}{10} \implies 1 = \dfrac{y}{10} \implies y = 10$

$r^2 = 10^2 + 10^2 \implies r = 10\sqrt{2}$

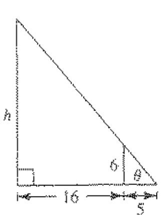
75. $\tan 45° = \dfrac{2\sqrt{5}}{x} \implies 1 = \dfrac{2\sqrt{5}}{x} \implies x = 2\sqrt{5}$

$r^2 = (2\sqrt{5})^2 + (2\sqrt{5})^2 = 20 + 20 = 40 \implies r = 2\sqrt{10}$

76. $\tan 45° = \dfrac{y}{4\sqrt{6}} \implies 1 = \dfrac{y}{4\sqrt{6}} \implies y = 4\sqrt{6}$

$r^2 = (4\sqrt{6})^2 + (4\sqrt{6})^2 = 96 + 96 = 192 \implies r = 8\sqrt{3}$

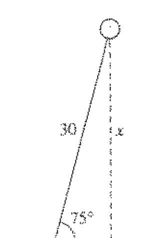
77. (a)



(b) $\tan \theta = \dfrac{6}{5}$ and $\tan \theta = \dfrac{h}{21}$. Thus, $\dfrac{6}{5} = \dfrac{h}{21}$.

(c) $h = \dfrac{6(21)}{5} = 25.2$ feet

78. (a)



(b) $\sin 75° = \dfrac{x}{30}$

(c) $x = 30 \sin 75° \approx 28.98$ meters

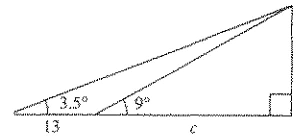
79. $\tan\theta = \dfrac{\text{opp}}{\text{adj}}$

$\tan 58^\circ = \dfrac{w}{100}$

$w = 100\tan 58^\circ \approx 160$ feet

80. $\cot 9^\circ = \dfrac{c}{h}$

$\cot 3.5^\circ = \dfrac{13 + c}{h}$



not drawn to scale

Subtracting, $\dfrac{13}{h} = \cot 3.5^\circ - \cot 9^\circ$

$h = \dfrac{13}{\cot 3.5^\circ - \cot 9^\circ}$

$\approx \dfrac{13}{16.3499 - 6.3138} \approx 1.295 \approx 1.3$ miles.

81. (a) $\tan\theta = \dfrac{50}{50} = 1 \implies \theta = 45^\circ$

(b) $L^2 = 50^2 + 50^2 = 2 \cdot 50^2 \implies L = 50\sqrt{2}$ feet

(c) $\dfrac{50\sqrt{2}}{6} = \dfrac{25\sqrt{2}}{3}$ ft/sec   rate down the zip line

$\dfrac{50}{6} = \dfrac{25}{3}$ ft/sec   vertical rate

82. (a) $\sin 35.4^\circ = \dfrac{x}{896.5}$

$x = 896.5 \sin 35.4^\circ \approx 519.3$ feet

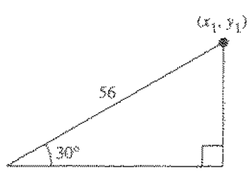
(b) $1693.5 - 519.3 = 1174.2$ feet above sea level

(c) $\dfrac{896.5}{300}$ minutes to reach top

Vertical rate $= \dfrac{519.3}{(896.5/300)}$

≈ 173.8 feet per minute

83.



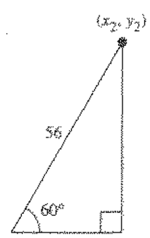
$\sin 30^\circ = \dfrac{y_1}{56}$

$y_1 = (\sin 30^\circ)(56) = \left(\dfrac{1}{2}\right)(56) = 28$

$\cos 30^\circ = \dfrac{x_1}{56}$

$x_1 = \cos 30^\circ(56) = \dfrac{\sqrt{3}}{2}(56) = 28\sqrt{3}$

$(x_1, y_1) = \left(28\sqrt{3}, 28\right)$



$\sin 60^\circ = \dfrac{y_2}{56}$

$y_2 = \sin 60^\circ(56) = \left(\dfrac{\sqrt{3}}{2}\right)(56) = 28\sqrt{3}$

$\cos 60^\circ = \dfrac{x_2}{56}$

$x_2 = (\cos 60^\circ)(56) = \left(\dfrac{1}{2}\right)(56) = 28$

$(x_2, y_2) = \left(28, 28\sqrt{3}\right)$

84. $x \approx 9.397,\ y \approx 3.420$

$$\sin 20^\circ = \frac{y}{10} \approx 0.34 \qquad \cos 20^\circ = \frac{x}{10} \approx 0.94$$

$$\tan 20^\circ = \frac{y}{x} \approx 0.36 \qquad \cot 20^\circ = \frac{x}{y} \approx 2.75$$

$$\sec 20^\circ = \frac{10}{x} \approx 1.06 \qquad \csc 20^\circ = \frac{10}{y} \approx 2.92$$

85. True

$$\sin 60^\circ \csc 60^\circ = \sin 60^\circ \frac{1}{\sin 60^\circ} = 1$$

86. False

$$\sin 45^\circ + \cos 45^\circ = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2} \neq 1$$

87. True

$1 + \cot^2\theta = \csc^2\theta$ for all θ

88. No. $\tan^2\theta + 1 = \sec^2\theta$, so you can find $\pm \sec\theta$.

89. (a)

θ	0°	20°	40°	60°	80°
$\sin\theta$	0	0.3420	0.6428	0.8660	0.9848
$\cos\theta$	1	0.9397	0.7660	0.5000	0.1736
$\tan\theta$	0	0.3640	0.8391	1.7321	5.6713

(b) Sine and tangent are increasing, cosine is decreasing.

(c) In each case, $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$.

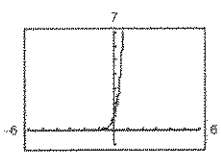
90.

θ	0°	20°	40°	60°	80°
$\cos\theta$	1	0.9397	0.7660	0.5000	0.1736
$\sin(90^\circ - \theta)$	1	0.9397	0.7660	0.5000	0.1736

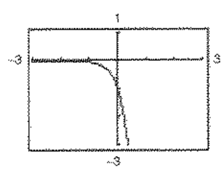
$\cos\theta = \sin(90^\circ - \theta)$

θ and $90^\circ - \theta$ are complementary angles.

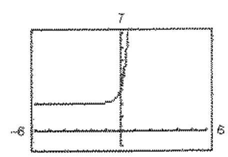
91.



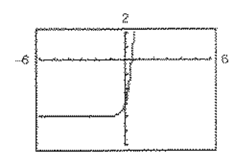
92.



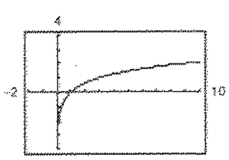
93.



94.



95.



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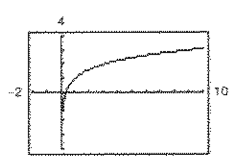
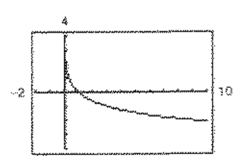
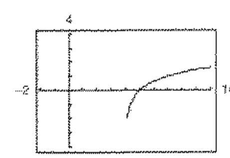


97.



98.



Section 4.4 Trigonometric Functions of Any Angle

- Know the Definitions of Trigonometric Functions of Any Angle.
  If θ is in standard position, (x, y) a point on the terminal side and $r = \sqrt{x^2 + y^2} \neq 0$, then:

  $\sin\theta = \dfrac{y}{r}$ $\qquad$ $\csc\theta = \dfrac{r}{y},\ y \neq 0$

  $\cos\theta = \dfrac{x}{r}$ $\qquad$ $\sec\theta = \dfrac{r}{x},\ x \neq 0$

  $\tan\theta = \dfrac{y}{x},\ x \neq 0$ $\qquad$ $\cot\theta = \dfrac{x}{y},\ y \neq 0$
- You should know the signs of the trigonometric functions in each quadrant.
- You should know the trigonometric function values of the quadrant angles $0, \dfrac{\pi}{2}, \pi$, and $\dfrac{3\pi}{2}$.
- You should be able to find reference angles.
- You should be able to evaluate trigonometric functions of any angle. (Use reference angles.)
- You should know that the period of sine and cosine is 2π.
- You should know which trigonometric functions are odd and even.
  Even: $\cos x$ and $\sec x$
  Odd: $\sin x$, $\tan x$, $\cot x$, $\csc x$

Vocabulary Check

1. $\dfrac{y}{r}$
2. $\csc\theta$
3. $\dfrac{y}{x}$
4. $\dfrac{r}{x}$
5. $\cos\theta$
6. $\cot\theta$
7. reference

1. (a) $(x, y) = (4, 3)$

$r = \sqrt{16 + 9} = 5$

$\sin\theta = \dfrac{y}{r} = \dfrac{3}{5}$ $\qquad$ $\csc\theta = \dfrac{r}{y} = \dfrac{5}{3}$

$\cos\theta = \dfrac{x}{r} = \dfrac{4}{5}$ $\qquad$ $\sec\theta = \dfrac{r}{x} = \dfrac{5}{4}$

$\tan\theta = \dfrac{y}{x} = \dfrac{3}{4}$ $\qquad$ $\cot\theta = \dfrac{x}{y} = \dfrac{4}{3}$

(b) $(x, y) = (-8, -15)$

$r = \sqrt{64 + 225} = 17$

$\sin\theta = \dfrac{y}{r} = -\dfrac{15}{17}$ $\qquad$ $\csc\theta = \dfrac{r}{y} = -\dfrac{17}{15}$

$\cos\theta = \dfrac{x}{r} = -\dfrac{8}{17}$ $\qquad$ $\sec\theta = \dfrac{r}{x} = -\dfrac{17}{8}$

$\tan\theta = \dfrac{y}{x} = \dfrac{15}{8}$ $\qquad$ $\cot\theta = \dfrac{x}{y} = \dfrac{8}{15}$

2. (a) $x = 12, y = -5$

$r = \sqrt{12^2 + (-5)^2} = 13$

$\sin\theta = \frac{y}{r} = \frac{-5}{13} = -\frac{5}{13}$

$\cos\theta = \frac{x}{r} = \frac{12}{13}$

$\tan\theta = \frac{y}{x} = \frac{-5}{12} = -\frac{5}{12}$

$\csc\theta = \frac{r}{y} = \frac{13}{-5} = -\frac{13}{5}$

$\sec\theta = \frac{r}{x} = \frac{13}{12}$

$\cot\theta = \frac{x}{y} = \frac{12}{-5} = -\frac{12}{5}$

(b) $x = -1, y = 1$

$r = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$

$\sin\theta = \frac{y}{r} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\cos\theta = \frac{x}{r} = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$

$\tan\theta = \frac{y}{x} = \frac{1}{-1} = -1$

$\csc\theta = \frac{r}{y} = \frac{\sqrt{2}}{1} = \sqrt{2}$

$\sec\theta = \frac{r}{x} = \frac{\sqrt{2}}{-1} = -\sqrt{2}$

$\cot\theta = \frac{x}{y} = \frac{-1}{1} = -1$

3. (a) $(x, y) = \left(-\sqrt{3}, -1\right)$

$r = \sqrt{3 + 1} = 2$

$\sin\theta = \frac{y}{r} = -\frac{1}{2}$ $\qquad \csc\theta = \frac{r}{y} = -2$

$\cos\theta = \frac{x}{r} = \frac{-\sqrt{3}}{2}$ $\qquad \sec\theta = \frac{r}{x} = \frac{-2\sqrt{3}}{3}$

$\tan\theta = \frac{y}{x} = \frac{\sqrt{3}}{3}$ $\qquad \cot\theta = \frac{x}{y} = \sqrt{3}$

(b) $(x, y) = (-2, 2)$

$r = \sqrt{4 + 4} = 2\sqrt{2}$

$\sin\theta = \frac{y}{r} = \frac{\sqrt{2}}{2}$ $\qquad \csc\theta = \frac{r}{y} = \sqrt{2}$

$\cos\theta = \frac{x}{r} = -\frac{\sqrt{2}}{2}$ $\qquad \sec\theta = \frac{r}{x} = -\sqrt{2}$

$\tan\theta = \frac{y}{x} = -1$ $\qquad \cot\theta = \frac{x}{y} = -1$

4. (a) $x = 3, y = 1$

$r = \sqrt{3^2 + 1^2} = \sqrt{10}$

$\sin\theta = \frac{y}{r} = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}$

$\cos\theta = \frac{x}{r} = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10}$

$\tan\theta = \frac{y}{x} = \frac{1}{3}$

$\csc\theta = \frac{r}{y} = \frac{\sqrt{10}}{1} = \sqrt{10}$

$\sec\theta = \frac{r}{x} = \frac{\sqrt{10}}{3}$

$\cot\theta = \frac{x}{y} = \frac{3}{1} = 3$

(b) $x = 2, y = -4$

$r = \sqrt{2^2 + (-4)^2} = 2\sqrt{5}$

$\sin\theta = \frac{y}{r} = \frac{-4}{2\sqrt{5}} = -\frac{2\sqrt{5}}{5}$

$\cos\theta = \frac{x}{r} = \frac{2}{2\sqrt{5}} = \frac{\sqrt{5}}{5}$

$\tan\theta = \frac{y}{x} = \frac{-4}{2} = -2$

$\csc\theta = \frac{r}{y} = \frac{2\sqrt{5}}{-4} = -\frac{\sqrt{5}}{2}$

$\sec\theta = \frac{r}{x} = \frac{2\sqrt{5}}{2} = \sqrt{5}$

$\cot\theta = \frac{x}{y} = \frac{2}{-4} = -\frac{1}{2}$

5. $(x, y) = (7, 24)$

$r = \sqrt{49 + 576} = 25$

$\sin\theta = \dfrac{y}{r} = \dfrac{24}{25}$ $\quad\csc\theta = \dfrac{r}{y} = \dfrac{25}{24}$

$\cos\theta = \dfrac{x}{r} = \dfrac{7}{25}$ $\quad\sec\theta = \dfrac{r}{x} = \dfrac{25}{7}$

$\tan\theta = \dfrac{y}{x} = \dfrac{24}{7}$ $\quad\cot\theta = \dfrac{x}{y} = \dfrac{7}{24}$

6. $x = 8, y = 15,$

$r = \sqrt{8^2 + 15^2} = 17$

$\sin\theta = \dfrac{y}{r} = \dfrac{15}{17}$ $\quad\cos\theta = \dfrac{x}{r} = \dfrac{8}{17}$

$\tan\theta = \dfrac{y}{x} = \dfrac{15}{8}$ $\quad\csc\theta = \dfrac{r}{y} = \dfrac{17}{15}$

$\sec\theta = \dfrac{r}{x} = \dfrac{17}{8}$ $\quad\cot\theta = \dfrac{x}{y} = \dfrac{8}{15}$

7. $(x, y) = (5, -12)$

$r = \sqrt{5^2 + (-12)^2} = \sqrt{25 + 144} = \sqrt{169} = 13$

$\sin\theta = \dfrac{y}{r} = -\dfrac{12}{13}$

$\cos\theta = \dfrac{x}{r} = \dfrac{5}{13}$

$\tan\theta = \dfrac{y}{x} = -\dfrac{12}{5}$

$\csc\theta = \dfrac{r}{y} = -\dfrac{13}{12}$

$\sec\theta = \dfrac{r}{x} = \dfrac{13}{5}$

$\cot\theta = \dfrac{x}{y} = -\dfrac{5}{12}$

8. $x = -24, y = 10$

$r = \sqrt{(-24)^2 + (10)^2} = 26$

$\sin\theta = \dfrac{y}{r} = \dfrac{10}{26} = \dfrac{5}{13}$

$\cos\theta = \dfrac{x}{r} = \dfrac{-24}{26} = \dfrac{-12}{13}$

$\tan\theta = \dfrac{y}{x} = \dfrac{10}{-24} = -\dfrac{5}{12}$

$\csc\theta = \dfrac{r}{y} = \dfrac{13}{5}$

$\sec\theta = \dfrac{r}{x} = -\dfrac{13}{12}$

$\cot\theta = \dfrac{x}{y} = -\dfrac{12}{5}$

9. $(x, y) = (-4, 10)$

$r = \sqrt{16 + 100} = 2\sqrt{29}$

$\sin\theta = \dfrac{y}{r} = \dfrac{5\sqrt{29}}{29}$

$\cos\theta = \dfrac{x}{r} = -\dfrac{2\sqrt{29}}{29}$

$\tan\theta = \dfrac{y}{x} = -\dfrac{5}{2}$

$\csc\theta = \dfrac{r}{y} = \dfrac{\sqrt{29}}{5}$

$\sec\theta = \dfrac{r}{x} = -\dfrac{\sqrt{29}}{2}$

$\cot\theta = \dfrac{x}{y} = -\dfrac{2}{5}$

10. $x = -5, y = -6$

$r = \sqrt{(-5)^2 + (-6)^2} = \sqrt{61}$

$\sin\theta = \dfrac{y}{r} = \dfrac{-6}{\sqrt{61}} = \dfrac{-6\sqrt{61}}{61}$

$\cos\theta = \dfrac{x}{r} = \dfrac{-5}{\sqrt{61}} = \dfrac{-5\sqrt{61}}{61}$

$\tan\theta = \dfrac{y}{x} = \dfrac{-6}{-5} = \dfrac{6}{5}$

$\csc\theta = \dfrac{r}{y} = -\dfrac{\sqrt{61}}{6}$

$\sec\theta = \dfrac{r}{x} = -\dfrac{\sqrt{61}}{5}$

$\cot\theta = \dfrac{x}{y} = \dfrac{5}{6}$

11. $(x, y) = (-10, 8)$

$r = \sqrt{(-10)^2 + 8^2} = \sqrt{164} = 2\sqrt{41}$

$\sin\theta = \frac{y}{r} = \frac{8}{2\sqrt{41}} = \frac{4\sqrt{41}}{41}$

$\cos\theta = \frac{x}{r} = \frac{-10}{2\sqrt{41}} = -\frac{5\sqrt{41}}{41}$

$\tan\theta = \frac{y}{x} = \frac{8}{-10} = -\frac{4}{5}$

$\csc\theta = \frac{r}{y} = \frac{\sqrt{41}}{4}$

$\sec\theta = \frac{r}{x} = -\frac{-\sqrt{41}}{5}$

$\cot\theta = \frac{x}{y} = -\frac{5}{4}$

12. $x = 3, y = -9$

$r = \sqrt{3^2 + (-9)^2} = \sqrt{90} = 3\sqrt{10}$

$\sin\theta = \frac{y}{r} = \frac{-9}{3\sqrt{10}} = \frac{-3\sqrt{10}}{10}$

$\cos\theta = \frac{x}{r} = \frac{3}{3\sqrt{10}} = \frac{\sqrt{10}}{10}$

$\tan\theta = \frac{y}{x} = \frac{-9}{3} = -3$

$\csc\theta = \frac{r}{y} = -\frac{\sqrt{10}}{3}$

$\sec\theta = \frac{r}{x} = \sqrt{10}$

$\cot\theta = \frac{x}{y} = -\frac{1}{3}$

13. $\sin\theta < 0 \implies \theta$ lies in Quadrant III or Quadrant IV.

$\cos\theta < 0 \implies \theta$ lies in Quadrant II or Quadrant III.

$\sin\theta < 0$ *and* $\cos\theta < 0 \implies \theta$ lies in Quadrant III.

14. $\sec\theta > 0$ and $\cot\theta < 0$

$\frac{r}{x} > 0$ and $\frac{x}{y} < 0$

Quadrant IV

15. $\cot\theta > 0 \implies \theta$ lies in Quadrant I or Quadrant III.

$\cos\theta > 0 \implies \theta$ lies in Quadrant I or Quadrant IV.

$\cot\theta > 0$ and $\cos\theta > 0 \implies \theta$ lies in Quadrant I.

16. $\tan\theta > 0$ and $\csc\theta < 0$

$\frac{y}{x} > 0$ and $\frac{r}{y} < 0$

Quadrant III

17. $\sin\theta = \frac{y}{r} = \frac{3}{5} \implies x^2 = 25 - 9 = 16$

θ in Quadrant II $\implies x = -4$

$\sin\theta = \frac{y}{r} = \frac{3}{5}$ $\qquad \csc\theta = \frac{r}{y} = \frac{5}{3}$

$\cos\theta = \frac{x}{r} = -\frac{4}{5}$ $\qquad \sec\theta = \frac{r}{x} = -\frac{5}{4}$

$\tan\theta = \frac{y}{x} = -\frac{3}{4}$ $\qquad \cot\theta = \frac{x}{y} = -\frac{4}{3}$

18. $\cos\theta = \frac{x}{r} = \frac{-4}{5} \implies |y| = 3$

θ in Quadrant III $\implies y = -3$

$\sin\theta = \frac{y}{r} = -\frac{3}{5}$ $\qquad \csc\theta = -\frac{5}{3}$

$\cos\theta = \frac{x}{r} = -\frac{4}{5}$ $\qquad \sec\theta = -\frac{5}{4}$

$\tan\theta = \frac{y}{x} = \frac{3}{4}$ $\qquad \cot\theta = \frac{4}{3}$

19. $\sin\theta < 0 \implies y < 0$

$\tan\theta = \frac{y}{x} = \frac{-15}{8} \implies r = 17$

$\sin\theta = \frac{y}{r} = -\frac{15}{17}$ $\quad\csc\theta = \frac{r}{y} = -\frac{17}{15}$

$\cos\theta = \frac{x}{r} = \frac{8}{17}$ $\quad\sec\theta = \frac{r}{x} = \frac{17}{8}$

$\cot\theta = \frac{x}{y} = -\frac{8}{15}$

20. $\csc\theta = \frac{r}{y} = \frac{4}{1} \implies x = \pm\sqrt{15}$

$\cot\theta < 0 \implies x = -\sqrt{15}$

$\sin\theta = \frac{y}{r} = \frac{1}{4}$ $\quad\csc\theta = 4$

$\cos\theta = \frac{x}{r} = -\frac{\sqrt{15}}{4}$ $\quad\sec\theta = -\frac{4\sqrt{15}}{15}$

$\tan\theta = \frac{y}{x} = -\frac{\sqrt{15}}{15}$ $\quad\cot\theta = -\sqrt{15}$

21. $\sec\theta = \frac{r}{x} = \frac{2}{-1} \implies y^2 = 4 - 1 = 3$

$\sin\theta \geq 0 \implies y = \sqrt{3}$

$\sin\theta = \frac{y}{r} = \frac{\sqrt{3}}{2}$ $\quad\csc\theta = \frac{r}{y} = \frac{2\sqrt{3}}{3}$

$\cos\theta = \frac{x}{r} = -\frac{1}{2}$ $\quad\sec\theta = \frac{r}{x} = -2$

$\tan\theta = \frac{y}{x} = -\sqrt{3}$ $\quad\cot\theta = \frac{x}{y} = -\frac{\sqrt{3}}{3}$

22. $\sin\theta = 0$ and $\frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2} \implies \theta = \pi$

$\cos\theta = \cos\pi = -1$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = 0$

$\csc\theta$ is undefined.

$\sec\theta = -1$

$\cot\theta$ is undefined.

23. $\cot\theta$ is undefined $\implies \theta = n\pi$.

$\frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2} \implies \theta = \pi, y = 0, x = -r$

$\sin\theta = \frac{y}{r} = 0$ $\quad\csc\theta = \frac{r}{y}$ is undefined.

$\cos\theta = \frac{x}{r} = \frac{-r}{r} = -1$ $\quad\sec\theta = \frac{r}{x} = -1$

$\tan\theta = \frac{y}{x} = \frac{0}{x} = 0$ $\quad\cot\theta$ is undefined.

24. $\tan\theta$ is undefined and $\pi \leq \theta \leq 2\pi \implies \theta = \frac{3\pi}{2}$.

$\sin\theta = -1$

$\cos\theta = 0$

$\tan\theta$ is undefined.

$\csc\theta = -1$

$\sec\theta$ is undefined.

$\cot\theta = 0$

25. To find a point on the terminal side of θ, use any point on the line $y = -x$ that lies in Quadrant II. $(-1, 1)$ is one such point.

$x = -1, y = 1, r = \sqrt{2}$

$\sin\theta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ $\quad\csc\theta = \sqrt{2}$

$\cos\theta = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$ $\quad\sec\theta = -\sqrt{2}$

$\tan\theta = -1$ $\quad\cot\theta = -1$

26. $\left(-x, -\frac{1}{3}x\right)$ Quadrant III, $x > 0$

$$r = \sqrt{x^2 + \frac{1}{9}x^2} = \frac{\sqrt{10}x}{3}$$

$$\sin\theta = \frac{y}{r} = \frac{(-x/3)}{(\sqrt{10}x)/3} = -\frac{\sqrt{10}}{10}$$

$$\cos\theta = \frac{x}{r} = \frac{-x}{(\sqrt{10}x)/3} = -\frac{3\sqrt{10}}{10}$$

$$\tan\theta = \frac{y}{x} = \frac{(-1/3)x}{-x} = \frac{1}{3}$$

$$\csc\theta = \frac{r}{y} = \frac{(\sqrt{10}x)/3}{(-1/3)x} = -\sqrt{10}$$

$$\sec\theta = \frac{r}{x} = \frac{(\sqrt{10}x)/3}{-x} = -\frac{\sqrt{10}}{3}$$

$$\cot\theta = \frac{x}{y} = \frac{-x}{(-1/3)x} = 3$$

27. To find a point on the terminal side of θ, use any point on the line $y = 2x$ that lies in Quadrant III. $(-1, -2)$ is one such point.

$x = -1, y = -2, r = \sqrt{5}$

$$\sin\theta = -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$$

$$\cos\theta = -\frac{1}{\sqrt{5}} = -\frac{\sqrt{5}}{5}$$

$$\tan\theta = \frac{-2}{-1} = 2$$

$$\csc\theta = \frac{\sqrt{5}}{-2} = -\frac{\sqrt{5}}{2}$$

$$\sec\theta = \frac{\sqrt{5}}{-1} = -\sqrt{5}$$

$$\cot\theta = \frac{-1}{-2} = \frac{1}{2}$$

28. $4x + 3y = 0 \implies y = -\frac{4}{3}x$

$\left(x, -\frac{4}{3}x\right)$ Quadrant IV, $x > 0$

$$r = \sqrt{x^2 + \frac{16}{9}x^2} = \frac{5}{3}x$$

$$\sin\theta = \frac{y}{r} = \frac{(-4/3)x}{(5/3)x} = -\frac{4}{5} \qquad \csc\theta = -\frac{5}{4}$$

$$\cos\theta = \frac{x}{r} = \frac{x}{(5/3)x} = \frac{3}{5} \qquad \sec\theta = \frac{5}{3}$$

$$\tan\theta = \frac{y}{x} = \frac{(-4/3)x}{x} = -\frac{4}{3} \qquad \cot\theta = -\frac{3}{4}$$

29. $(x, y) = (-1, 0)$

$$\sec\pi = \frac{r}{x} = \frac{1}{-1} = -1$$

30. $\tan\frac{\pi}{2} = \frac{y}{x} = \frac{1}{0} \implies$ undefined

since $\frac{\pi}{2}$ corresponds to $(0, 1)$.

31. $(x, y) = (0, -1)$

$$\cot\left(\frac{3\pi}{2}\right) = \frac{x}{y} = \frac{0}{-1} = 0$$

32. $\csc 0 = \frac{r}{y} = \frac{1}{0} \implies$ undefined

33. $(x, y) = (1, 0)$

$$\sec 0 = \frac{r}{x} = \frac{1}{1} = 1$$

34. $\csc\frac{3\pi}{2} = \frac{1}{\sin\frac{3\pi}{2}} = \frac{1}{-1} = -1$

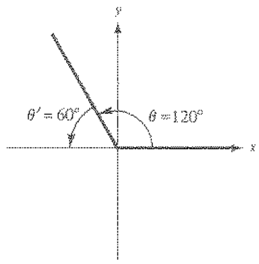
35. $(x, y) = (-1, 0)$

$$\cot\pi = \frac{x}{y} = -\frac{1}{0} \implies \text{undefined}$$

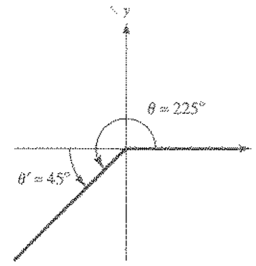
36. $\csc\frac{\pi}{2} = \frac{1}{\sin\frac{\pi}{2}} = \frac{1}{1} = 1$

37. $\theta = 120^\circ$

$\theta' = 180^\circ - 120^\circ = 60^\circ$

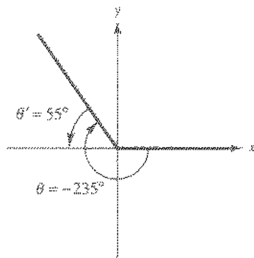


38. $\theta' = 225^\circ - 180^\circ = 45^\circ$

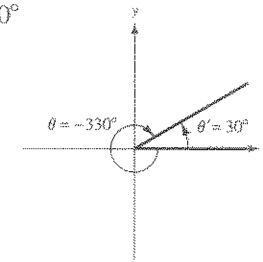


39. $\theta = -135^\circ$ is coterminal with 225°.

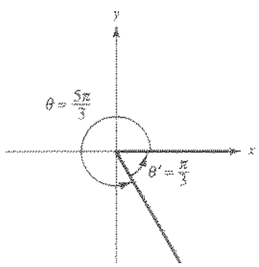
$\theta' = 225^\circ - 180^\circ = 45^\circ$



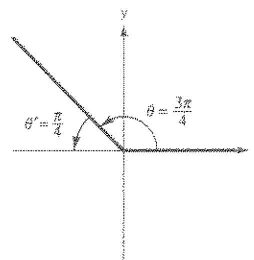
40. $\theta' = -330^\circ + 360^\circ = 30^\circ$



41. $\theta = \dfrac{5\pi}{3}, \theta' = 2\pi - \dfrac{5\pi}{3} = \dfrac{\pi}{3}$

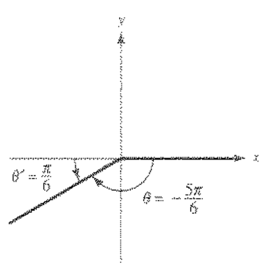


42. $\theta' = \pi - \dfrac{3\pi}{4} = \dfrac{\pi}{4}$

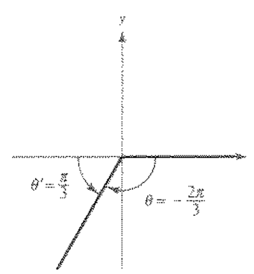


43. $\theta = -\dfrac{5\pi}{6}$ is coterminal with $\dfrac{7\pi}{6}$.

$\theta' = \dfrac{7\pi}{6} - \pi = \dfrac{\pi}{6}$

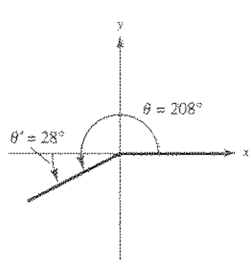


44. $\theta' = \dfrac{-2\pi}{3} + \pi = \dfrac{\pi}{3}$



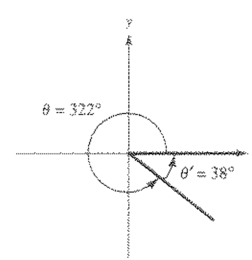
45. $\theta = 208^\circ$

$\theta' = 208^\circ - 180^\circ = 28^\circ$



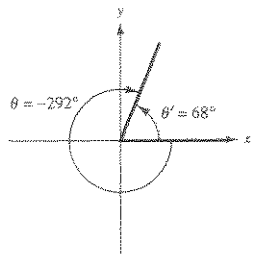
46. $\theta = 322^\circ$

$\theta' = 360^\circ - 322^\circ = 38^\circ$



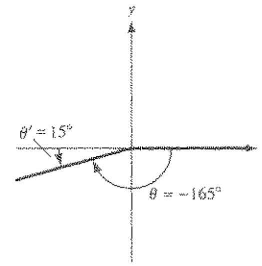
47. $\theta = -292^\circ$

$\theta' = 360^\circ - 292^\circ = 68^\circ$



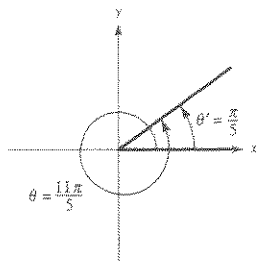
48. $\theta = -165^\circ$ lies in Quadrant III.

Reference angle:
$180^\circ - 165^\circ = 15^\circ$

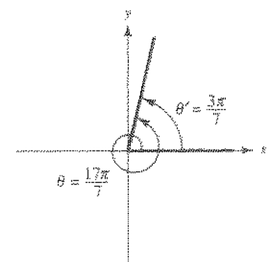


49. $\theta = \dfrac{11\pi}{5}$ is coterminal with $\dfrac{\pi}{5}$.

$\theta' = \dfrac{\pi}{5}$

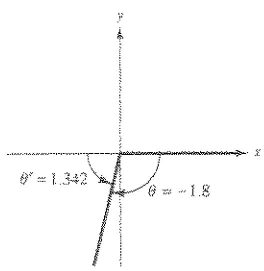


50. $\theta' = \dfrac{17\pi}{7} - \dfrac{14\pi}{7} = \dfrac{3\pi}{7}$



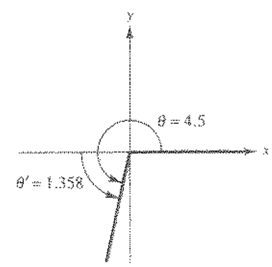
51. $\theta = -1.8$ lies in Quadrant III.

Reference angle: $\pi - 1.8 \approx 1.342$



52. θ lies in Quadrant IV.

Reference angle: $4.5 - \pi \approx 1.358$



53. $\theta' = 45^\circ$, Quadrant III

$\sin 225^\circ = -\sin 45^\circ = -\dfrac{\sqrt{2}}{2}$

$\cos 225^\circ = -\cos 45^\circ = -\dfrac{\sqrt{2}}{2}$

$\tan 225^\circ = \tan 45^\circ = 1$

54. $\theta = 300^\circ$, $\theta' = 360^\circ - 300^\circ = 60^\circ$, Quadrant IV

$\sin 300^\circ = -\sin 60^\circ = -\dfrac{\sqrt{3}}{2}$

$\cos 300^\circ = \cos 60^\circ = \dfrac{1}{2}$

$\tan 300^\circ = -\tan 60^\circ = -\sqrt{3}$

55. $\theta = -750^\circ$ is coterminal with 330°, Quadrant IV.

$\theta' = 360^\circ - 330^\circ = 30^\circ$

$\sin(-750^\circ) = -\sin 30^\circ = -\dfrac{1}{2}$

$\cos(-750^\circ) = \cos 30^\circ = \dfrac{\sqrt{3}}{2}$

$\tan(-750^\circ) = -\tan 30^\circ = -\dfrac{\sqrt{3}}{3}$

56. $\theta = -495^\circ$, $\theta' = 45^\circ$, Quadrant III

$\sin(-495^\circ) = -\sin 45^\circ = -\dfrac{\sqrt{2}}{2}$

$\cos(-495^\circ) = -\cos 45^\circ = -\dfrac{\sqrt{2}}{2}$

$\tan(-495^\circ) = \tan 45^\circ = 1$

57. $\theta = \frac{5\pi}{3}$, Quadrant IV

$\theta' = 2\pi - \frac{5\pi}{3} = \frac{\pi}{3}$

$\sin\left(\frac{5\pi}{3}\right) = -\sin\left(\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$

$\cos\left(\frac{5\pi}{3}\right) = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

$\tan\left(\frac{5\pi}{3}\right) = -\tan\left(\frac{\pi}{3}\right) = -\sqrt{3}$

58. $\theta = \frac{3\pi}{4}, \theta' = \pi - \frac{3\pi}{4} = \frac{\pi}{4}$

$\sin\frac{3\pi}{4} = \frac{\sqrt{2}}{2}$

$\cos\frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$

$\tan\frac{3\pi}{4} = -1$

59. $\theta' = \frac{\pi}{6}$, Quadrant IV

$\sin\left(-\frac{\pi}{6}\right) = -\sin\frac{\pi}{6} = -\frac{1}{2}$

$\cos\left(-\frac{\pi}{6}\right) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

$\tan\left(-\frac{\pi}{6}\right) = -\tan\frac{\pi}{6} = -\frac{\sqrt{3}}{3}$

60. $\theta = \frac{-4\pi}{3}, \theta' = \frac{\pi}{3}$

$\sin\left(\frac{-4\pi}{3}\right) = \frac{\sqrt{3}}{2}$

$\cos\left(\frac{-4\pi}{3}\right) = \frac{-1}{2}$

$\tan\left(\frac{-4\pi}{3}\right) = -\sqrt{3}$

61. $\theta' = \frac{\pi}{4}$, Quadrant II

$\sin\frac{11\pi}{4} = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$\cos\frac{11\pi}{4} = -\cos\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$

$\tan\frac{11\pi}{4} = -\tan\frac{\pi}{4} = -1$

62. $\theta = \frac{10\pi}{3}$ is coterminal with $\frac{4\pi}{3}$.

$\theta' = \frac{4\pi}{3} - \pi = \frac{\pi}{3}$ in Quadrant III.

$\sin\frac{10\pi}{3} = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$

$\cos\frac{10\pi}{3} = -\cos\frac{\pi}{3} = -\frac{1}{2}$

$\tan\frac{10\pi}{3} = \tan\frac{\pi}{3} = \sqrt{3}$

63. $\theta = -\frac{17\pi}{6}$ is coterminal with $\frac{7\pi}{6}$.

$\theta' = \frac{7\pi}{6} - \pi = \frac{\pi}{6}$, Quadrant III

$\sin\left(-\frac{17\pi}{6}\right) = -\sin\left(\frac{\pi}{6}\right) = -\frac{1}{2}$

$\cos\left(-\frac{17\pi}{6}\right) = -\cos\left(\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{2}$

$\tan\left(-\frac{17\pi}{6}\right) = \tan\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{3}$

64. $\theta = \frac{-20\pi}{3}, \theta' = \frac{\pi}{3}$

$\sin\left(\frac{-20\pi}{3}\right) = \frac{-\sqrt{3}}{2}$

$\cos\left(\frac{-20\pi}{3}\right) = \frac{-1}{2}$

$\tan\left(\frac{-20\pi}{3}\right) = \sqrt{3}$

65. $\sin\theta = -\frac{3}{5}$

$\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

$\cos^2\theta = 1 - \left(-\frac{3}{5}\right)^2$

$\cos^2\theta = 1 - \frac{9}{25}$

$\cos^2\theta = \frac{16}{25}$

$\cos\theta > 0$ in Quadrant IV.

$\cos\theta = \frac{4}{5}$

66. $\cot\theta = -3$

$1 + \cot^2\theta = \csc^2\theta$

$1 + (-3)^2 = \csc^2\theta$

$10 = \csc^2\theta$

$\csc\theta > 0$ in Quadrant II.

$\sqrt{10} = \csc\theta$

$\csc\theta = \frac{1}{\sin\theta}$

$\sin\theta = \frac{1}{\csc\theta} = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}$

67. $\csc\theta = -2$

$1 + \cot^2\theta = \csc^2\theta$

$\cot^2\theta = \csc^2\theta - 1$

$\cot^2\theta = (-2)^2 - 1$

$\cot^2\theta = 3$

$\cot\theta < 0$ in Quadrant IV.

$\cot\theta = -\sqrt{3}$

68. $\cos\theta = \frac{5}{8}$

$\cos\theta = \frac{1}{\sec\theta} \implies \sec\theta = \frac{1}{\cos\theta}$

$\sec\theta = \frac{1}{5/8} = \frac{8}{5}$

69. $\sec\theta = -\frac{9}{4}$

$1 + \tan^2\theta = \sec^2\theta$

$\tan^2\theta = \sec^2\theta - 1$

$\tan^2\theta = \left(-\frac{9}{4}\right)^2 - 1$

$\tan^2\theta = \frac{65}{16}$

$\tan\theta > 0$ in Quadrant III.

$\tan\theta = \frac{\sqrt{65}}{4}$

70. $\tan\theta = -\frac{5}{4} \implies \cot\theta = -\frac{4}{5}$

$1 + \cot^2\theta = \csc^2\theta$

$1 + \frac{16}{25} = \csc^2\theta$

$\csc\theta = \pm\frac{\sqrt{41}}{5}$

Quadrant IV $\implies \csc\theta = -\frac{\sqrt{41}}{5}$

71. $\sin\theta = \frac{2}{5}$ and $\cos\theta < 0 \Longrightarrow \theta$ is in Quadrant II.

$$\cos\theta = -\sqrt{1-\sin^2\theta} = -\sqrt{1-\frac{4}{25}} = -\frac{\sqrt{21}}{5}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{2/5}{-\sqrt{21}/5} = \frac{-2}{\sqrt{21}} = \frac{-2\sqrt{21}}{21}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{5}{2}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{-5}{\sqrt{21}} = \frac{-5\sqrt{21}}{21}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{-\sqrt{21}}{2}$$

72. $\cos\theta = -\frac{3}{7}$ and $\sin\theta < 0 \Longrightarrow \theta$ is in Quadrant III.

$$\sin\theta = -\sqrt{1-\cos^2\theta} = -\sqrt{1-\frac{9}{49}} = \frac{-\sqrt{40}}{7} = \frac{-2\sqrt{10}}{7}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-2\sqrt{10}/7}{-3/7} = \frac{2\sqrt{10}}{3}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{3}{2\sqrt{10}} = \frac{3\sqrt{10}}{20}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{-7}{2\sqrt{10}} = \frac{-7\sqrt{10}}{20}$$

$$\sec\theta = \frac{1}{\cos\theta} = -\frac{7}{3}$$

73. $\tan\theta = -4$ and $\cos\theta < 0 \Longrightarrow \theta$ is in Quadrant II.

$$\sec\theta = -\sqrt{1+\tan^2\theta} = -\sqrt{1+16} = -\sqrt{17}$$

$$\cos\theta = \frac{1}{\sec\theta} = \frac{-1}{\sqrt{17}} = \frac{-\sqrt{17}}{17}$$

$$\sin\theta = \tan\theta\cos\theta = (-4)\left(-\frac{\sqrt{17}}{17}\right) = \frac{4\sqrt{17}}{17}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{17}{4\sqrt{17}} = \frac{\sqrt{17}}{4}$$

$$\cot\theta = \frac{1}{\tan\theta} = -\frac{1}{4}$$

74. $\cot\theta = -5$ and $\sin\theta > 0 \Longrightarrow \theta$ is in Quadrant II.

$$\tan\theta = \frac{1}{\cot\theta} = \frac{-1}{5}$$

$$\sec\theta = -\sqrt{1+\tan^2\theta} = -\sqrt{1+\frac{1}{25}} = \frac{-\sqrt{26}}{5}$$

$$\cos\theta = \frac{1}{\sec\theta} = \frac{-5}{\sqrt{26}} = \frac{-5\sqrt{26}}{26}$$

$$\sin\theta = \tan\theta\cos\theta = \left(-\frac{1}{5}\right)\left(\frac{-5\sqrt{26}}{26}\right) = \frac{\sqrt{26}}{26}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{26}{\sqrt{26}} = \sqrt{26}$$

75. $\csc\theta = -\frac{3}{2}$ and $\tan\theta < 0 \Longrightarrow \theta$ is in Quadrant IV.

$$\sin\theta = \frac{1}{\csc\theta} = -\frac{2}{3}$$

$$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\frac{4}{9}} = \frac{\sqrt{5}}{3}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{3}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-2/3}{\sqrt{5}/3} = \frac{-2}{\sqrt{5}} = \frac{-2\sqrt{5}}{5}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{-\sqrt{5}}{2}$$

76. $\sec\theta = -\frac{4}{3}$ and $\cot\theta > 0 \Longrightarrow \theta$ is in Quadrant III.

$$\cos\theta = \frac{1}{\sec\theta} = -\frac{3}{4}$$

$$\sin\theta = -\sqrt{1-\cos^2\theta} = -\sqrt{1-\frac{9}{16}} = -\frac{\sqrt{7}}{4}$$

$$\csc\theta = \frac{1}{\sin\theta} = -\frac{4}{\sqrt{7}} = \frac{-4\sqrt{7}}{7}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-\sqrt{7}/4}{-3/4} = \frac{\sqrt{7}}{3}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{3}{\sqrt{7}} = \frac{3\sqrt{7}}{7}$$

77. $\sin 10° \approx 0.1736$

78. $\sec 235° = \frac{1}{\cos 235°} \approx -1.7434$

79. $\tan 245° \approx 2.1445$

80. $\csc 320° = \frac{1}{\sin 320°} = -1.5557$

81. $\cos(-110°) \approx -0.3420$

82. $\cot(-220°) = \frac{1}{\tan(-220°)} \approx -1.1918$

83. $\sec(-280°) = \frac{1}{\cos(-280°)} \approx 5.7588$

84. $\csc 0.33 = \frac{1}{\sin 0.33} \approx 3.0860$

85. $\tan\left(\frac{2\pi}{9}\right) \approx 0.8391$

86. $\tan\frac{11\pi}{9} \approx 0.8391$

87. $\csc\left(-\frac{8\pi}{9}\right) = \frac{1}{\sin\left(-\frac{8\pi}{9}\right)} \approx -2.9238$

88. $\cos\left(\frac{-15\pi}{14}\right) \approx -0.9749$

89. (a) $\sin\theta = \frac{1}{2} \Longrightarrow$ reference angle is 30° or $\frac{\pi}{6}$ and θ is in Quadrant I or Quadrant II.

Values in degrees: 30°, 150°

Values in radian: $\frac{\pi}{6}, \frac{5\pi}{6}$

(b) $\sin\theta = -\frac{1}{2} \Longrightarrow$ reference angle is 30° or $\frac{\pi}{6}$ and θ is in Quadrant III or Quadrant IV.

Values in degrees: 210°, 330°

Values in radians: $\frac{7\pi}{6}, \frac{11\pi}{6}$

90. (a) $\cos\theta = \frac{\sqrt{2}}{2} \Longrightarrow$ reference angle is 45° or $\frac{\pi}{4}$ and θ is in Quadrant I or IV.

Values in degrees: 45°, 315°

Values in radians: $\frac{\pi}{4}, \frac{7\pi}{4}$

(b) $\cos\theta = -\frac{\sqrt{2}}{2} \Longrightarrow$ reference angle is 45° or $\frac{\pi}{4}$ and θ is in Quadrant II or III.

Values in degrees: 135°, 225°

Values in radians: $\frac{3\pi}{4}, \frac{5\pi}{4}$

91. (a) $\csc \theta = \frac{2\sqrt{3}}{3} \implies$ reference angle is $60°$ or $\frac{\pi}{3}$ and θ is in Quadrant I or Quadrant II.

Values in degrees: $60°, 120°$

Values in radians: $\frac{\pi}{3}, \frac{2\pi}{3}$

(b) $\cot \theta = -1 \implies$ reference angle is $45°$ or $\frac{\pi}{4}$ and θ is in Quadrant II or Quadrant IV.

Values in degrees: $135°, 315°$

Values in radians: $\frac{3\pi}{4}, \frac{7\pi}{4}$

92. (a) $\csc \theta = -\sqrt{2} \implies \sin \theta = \frac{-1}{\sqrt{2}}$

Reference angle is $45°$ or $\frac{\pi}{4}$.

Values in degrees: $225°, 315°$

Values in radians: $\frac{5\pi}{4}, \frac{7\pi}{4}$

(b) $\csc \theta = 2 \implies \sin \theta = \frac{1}{2}$

Reference angle is $\frac{\pi}{6}$ or $30°$.

Values in degrees: $30°, 150°$

Values in radians: $\frac{\pi}{6}, \frac{5\pi}{6}$

93. (a) $\sec \theta = -\frac{2\sqrt{3}}{3} \implies$ reference angle is $\frac{\pi}{6}$ or $30°$, and θ is in Quadrant II or Quadrant III.

Values in degrees: $150°, 210°$

Values in radians: $\frac{5\pi}{6}, \frac{7\pi}{6}$

(b) $\cos \theta = -\frac{1}{2} \implies$ reference angle is $\frac{\pi}{3}$ or $60°$, and θ is in Quadrant II or Quadrant III.

Values in degrees: $120°, 240°$

Values in radians: $\frac{2\pi}{3}, \frac{4\pi}{3}$

94. (a) $\cot \theta = -\sqrt{3} \implies \frac{\cos \theta}{\sin \theta} = -\sqrt{3}$

Reference angle is $\frac{\pi}{6}$ or $30°$.

Values in degrees: $150°, 330°$

Values in radians: $\frac{5\pi}{6}, \frac{11\pi}{6}$

(b) Values in degrees: $45°$ or $315°$

Values in radians: $\frac{\pi}{4}$ or $\frac{7\pi}{4}$

95. (a) $f(\theta) + g(\theta) = \sin 30° + \cos 30° = \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2}$

(b) $\cos 30° - \sin 30° = \frac{\sqrt{3}-1}{2}$

(c) $[\cos 30°]^2 = \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4}$

(d) $\sin 30° \cos 30° = \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{4}$

(e) $2 \sin 30° = 2\left(\frac{1}{2}\right) = 1$

(f) $\cos(-30°) = \cos 30° = \frac{\sqrt{3}}{2}$

96. (a) $f(\theta) + g(\theta) = \sin 60° + \cos 60° = \frac{\sqrt{3}}{2} + \frac{1}{2} = \frac{\sqrt{3}+1}{2}$

(b) $\cos 60° - \sin 60° = \frac{1}{2} - \frac{\sqrt{3}}{2} = \frac{1-\sqrt{3}}{2}$

(c) $[\cos 60°]^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

(d) $\sin 60° \cos 60° = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{4}$

(e) $2 \sin 60° = 2\frac{\sqrt{3}}{2} = \sqrt{3}$

(f) $\cos(-60°) = \cos 60° = \frac{1}{2}$

97. (a) $f(\theta) + g(\theta) = \sin 315° + \cos 315° = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = 0$

(b) $\cos 315° - \sin 315° = \frac{\sqrt{2}}{2} - \left(\frac{-\sqrt{2}}{2}\right) = \sqrt{2}$

(c) $[\cos 315°]^2 = \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{1}{2}$

(d) $\sin 315° \cos 315° = \left(\frac{-\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = -\frac{1}{2}$

(e) $2 \sin 315° = 2\left(\frac{-\sqrt{2}}{2}\right) = -\sqrt{2}$

(f) $\cos(-315°) = \cos(315°) = \frac{\sqrt{2}}{2}$

98. (a) $f(\theta) + g(\theta) = \sin 225° + \cos 225° = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} = -\sqrt{2}$

(b) $\cos 225° - \sin 225° = \frac{-\sqrt{2}}{2} - \left(\frac{-\sqrt{2}}{2}\right) = 0$

(c) $[\cos 225°]^2 = \left(\frac{-\sqrt{2}}{2}\right)^2 = \frac{1}{2}$

(d) $\sin 225° \cos 225° = \left(\frac{-\sqrt{2}}{2}\right)\left(\frac{-\sqrt{2}}{2}\right) = \frac{1}{2}$

(e) $2 \sin 225° = 2\left(\frac{-\sqrt{2}}{2}\right) = -\sqrt{2}$

(f) $\cos(-225°) = \cos(225°) = -\frac{\sqrt{2}}{2}$

99. (a) $f(\theta) + g(\theta) = \sin 150° + \cos 150° = \frac{1}{2} + \frac{-\sqrt{3}}{2} = \frac{1 - \sqrt{3}}{2}$

(b) $\cos 150° - \sin 150° = \frac{-\sqrt{3}}{2} - \frac{1}{2} = \frac{-1 - \sqrt{3}}{2}$

(c) $[\cos 150°]^2 = \left(\frac{-\sqrt{3}}{2}\right)^2 = \frac{3}{4}$

(d) $\sin 150° \cos 150° = \frac{1}{2} \cdot \frac{-\sqrt{3}}{2} = \frac{-\sqrt{3}}{4}$

(e) $2 \sin 150° = 2\left(\frac{1}{2}\right) = 1$

(f) $\cos(-150°) = \cos(150°) = \frac{-\sqrt{3}}{2}$

100. (a) $f(\theta) + g(\theta) = \sin 300° + \cos 300° = \frac{-\sqrt{3}}{2} + \frac{1}{2} = \frac{1 - \sqrt{3}}{2}$

(b) $\cos 300° - \sin 300° = \frac{1}{2} - \left(\frac{-\sqrt{3}}{2}\right) = \frac{1 + \sqrt{3}}{2}$

(c) $[\cos 300°]^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

(d) $\sin 300° \cos 300° = \left(\frac{-\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) = \frac{-\sqrt{3}}{4}$

(e) $2 \sin 300° = 2\left(\frac{-\sqrt{3}}{2}\right) = -\sqrt{3}$

(f) $\cos(-300°) = \cos(300°) = \frac{1}{2}$

101. (a) $f(\theta) + g(\theta) = \sin \frac{7\pi}{6} + \cos \frac{7\pi}{6} = -\frac{1}{2} - \frac{\sqrt{3}}{2} = \frac{-1 - \sqrt{3}}{2}$

(b) $\cos \frac{7\pi}{6} - \sin \frac{7\pi}{6} = \frac{-\sqrt{3}}{2} - \left(-\frac{1}{2}\right) = \frac{1 - \sqrt{3}}{2}$

(c) $\left[\cos \frac{7\pi}{6}\right]^2 = \left(\frac{-\sqrt{3}}{2}\right)^2 = \frac{3}{4}$

(d) $\sin \frac{7\pi}{6} \cos \frac{7\pi}{6} = \left(-\frac{1}{2}\right)\left(-\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{4}$

(e) $2 \sin \frac{7\pi}{6} = 2\left(-\frac{1}{2}\right) = -1$

(f) $\cos\left(\frac{-7\pi}{6}\right) = \cos\left(\frac{7\pi}{6}\right) = \frac{-\sqrt{3}}{2}$

102. (a) $f(\theta) + g(\theta) = \sin\frac{5\pi}{6} + \cos\frac{5\pi}{6} = \frac{1}{2} + \frac{-\sqrt{3}}{2} = \frac{1-\sqrt{3}}{2}$

(b) $\cos\frac{5\pi}{6} - \sin\frac{5\pi}{6} = \frac{-\sqrt{3}}{2} - \frac{1}{2} = \frac{-1-\sqrt{3}}{2}$

(c) $\left[\cos\frac{5\pi}{6}\right]^2 = \left(\frac{-\sqrt{3}}{2}\right)^2 = \frac{3}{4}$

(d) $\sin\frac{5\pi}{6}\cos\frac{5\pi}{6} = \left(\frac{1}{2}\right)\left(\frac{-\sqrt{3}}{2}\right) = \frac{-\sqrt{3}}{4}$

(e) $2\sin\frac{5\pi}{6} = 2\left(\frac{1}{2}\right) = 1$

(f) $\cos\left(\frac{-5\pi}{6}\right) = \cos\left(\frac{5\pi}{6}\right) = \frac{-\sqrt{3}}{2}$

103. (a) $f(\theta) + g(\theta) = \sin\frac{4\pi}{3} + \cos\frac{4\pi}{3} = \frac{-\sqrt{3}}{2} - \frac{1}{2} = \frac{-1-\sqrt{3}}{2}$

(b) $\cos\frac{4\pi}{3} - \sin\frac{4\pi}{3} = -\frac{1}{2} - \left(\frac{-\sqrt{3}}{2}\right) = \frac{\sqrt{3}-1}{2}$

(c) $\left[\cos\frac{4\pi}{3}\right]^2 = \left(-\frac{1}{2}\right)^2 = \frac{1}{4}$

(d) $\sin\frac{4\pi}{3}\cos\frac{4\pi}{3} = \left(\frac{-\sqrt{3}}{2}\right)\left(-\frac{1}{2}\right) = \frac{\sqrt{3}}{4}$

(e) $2\sin\frac{4\pi}{3} = 2\left(\frac{-\sqrt{3}}{2}\right) = -\sqrt{3}$

(f) $\cos\left(\frac{-4\pi}{3}\right) = \cos\left(\frac{4\pi}{3}\right) = -\frac{1}{2}$

104. (a) $f(\theta) + g(\theta) = \sin\frac{5\pi}{3} + \cos\frac{5\pi}{3} = \frac{-\sqrt{3}}{2} + \frac{1}{2} = \frac{1-\sqrt{3}}{2}$

(b) $\cos\frac{5\pi}{3} - \sin\frac{5\pi}{3} = \frac{1}{2} - \left(\frac{-\sqrt{3}}{2}\right) = \frac{1+\sqrt{3}}{2}$

(c) $\left[\cos\frac{5\pi}{3}\right]^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

(d) $\sin\frac{5\pi}{3}\cos\frac{5\pi}{3} = \left(\frac{-\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) = \frac{-\sqrt{3}}{4}$

(e) $2\sin\frac{5\pi}{3} = 2\left(\frac{-\sqrt{3}}{2}\right) = -\sqrt{3}$

(f) $\cos\left(\frac{-5\pi}{3}\right) = \cos\left(\frac{5\pi}{3}\right) = \frac{1}{2}$

105. (a) $f(\theta) + g(\theta) = \sin 270^\circ + \cos 270^\circ = -1 + 0 = -1$

(b) $\cos 270^\circ - \sin 270^\circ = 0 - (-1) = 1$

(c) $[\cos 270^\circ]^2 = 0^2 = 0$

(d) $\sin 270^\circ \cos 270^\circ = (-1)(0) = 0$

(e) $2\sin 270^\circ = 2(-1) = -2$

(f) $\cos(-270^\circ) = \cos(270^\circ) = 0$

106. (a) $f(\theta) + g(\theta) = \sin 180^\circ + \cos 180^\circ = 0 - 1 = -1$

(b) $\cos 180^\circ - \sin 180^\circ = -1 - 0 = -1$

(c) $[\cos 180^\circ]^2 = (-1)^2 = 1$

(d) $\sin 180^\circ \cos 180^\circ = 0(-1) = 0$

(e) $2\sin 180^\circ = 2(0) = 0$

(f) $\cos(-180^\circ) = \cos(180^\circ) = -1$

107. (a) $f(\theta) + g(\theta) = \sin\frac{7\pi}{2} + \cos\frac{7\pi}{2} = -1 + 0 = -1$

(b) $\cos\frac{7\pi}{2} - \sin\frac{7\pi}{2} = 0 - (-1) = 1$

(c) $\left[\cos\frac{7\pi}{2}\right]^2 = 0^2 = 0$

(d) $\sin\frac{7\pi}{2}\cos\frac{7\pi}{2} = (-1)(0) = 0$

(e) $2\sin\frac{7\pi}{2} = 2(-1) = -2$

(f) $\cos\left(\frac{-7\pi}{2}\right) = \cos\left(\frac{7\pi}{2}\right) = 0$

108. (a) $f(\theta) + g(\theta) = \sin\frac{5\pi}{2} + \cos\frac{5\pi}{2} = 1 + 0 = 1$

(b) $\cos\frac{5\pi}{2} - \sin\frac{5\pi}{2} = 0 - 1 = -1$

(c) $\left[\cos\frac{5\pi}{2}\right]^2 = 0^2 = 0$

(d) $\sin\frac{5\pi}{2}\cos\frac{5\pi}{2} = (1)(0) = 0$

(e) $2\sin\frac{5\pi}{2} = 2(1) = 2$

(f) $\cos\left(\frac{-5\pi}{2}\right) = \cos\left(\frac{5\pi}{2}\right) = 0$

109. $T = 49.5 + 20.5\cos\left(\frac{\pi t}{6} - \frac{7\pi}{6}\right)$

(a) January: $t = 1 \implies T = 49.5 + 20.5\cos\left(\frac{\pi(1)}{6} - \frac{7\pi}{6}\right) = 29^\circ$

(b) July: $t = 7 \implies T = 70^\circ$

(c) December: $t = 12 \implies T \approx 31.75^\circ$

110. $S = 23.1 + 0.442t + 4.3\sin\left(\frac{\pi t}{6}\right)$, $t = 1 \leftrightarrow$ Jan. 2004

(a) $S(1) = 23.1 + 0.442(1) + 4.3\sin\left(\frac{\pi}{6}\right) \approx 25.7$ thousand

(b) $S(14) \approx 33.0$ thousand

(c) $S(5) \approx 27.5$ thousand

(d) $S(6) \approx 25.8$ thousand

Answers will vary.

111. $\sin\theta = \frac{6}{d} \implies d = \frac{6}{\sin\theta}$

(a) $\theta = 30^\circ$

$$d = \frac{6}{\sin 30^\circ} = \frac{6}{(1/2)} = 12 \text{ miles}$$

(b) $\theta = 90^\circ$

$$d = \frac{6}{\sin 90^\circ} = \frac{6}{1} = 6 \text{ miles}$$

(c) $\theta = 120^\circ$

$$d = \frac{6}{\sin 120^\circ} \approx 6.9 \text{ miles}$$

112. As θ increases from 0° to 90°, x decreases from 12 cm to 0 cm and y increases from 0 cm to 12 cm. Therefore, $\sin\theta = y/12$ increases from 0 to 1, and $\cos\theta = x/12$ decreases from 1 to 0. Thus, $\tan\theta = y/x$ begins at 0 and increases without bound. When $\theta = 90^\circ$, the tangent is undefined.

113. True. The reference angle for $\theta = 151^\circ$ is $\theta' = 180^\circ - 151^\circ = 29^\circ$, and sine is positive in Quadrants I and II.

114. False. $-\cot\left(\frac{3\pi}{4}\right) = -(-1) = 1$ and

$\cot\left(-\frac{\pi}{4}\right) = -1$

115. (a)

θ	$0°$	$20°$	$40°$	$60°$	$80°$
$\sin\theta$	0	0.3420	0.6428	0.8660	0.9848
$\sin(180° - \theta)$	0	0.3420	0.6428	0.8660	0.9848

(b) It appears that $\sin\theta = \sin(180° - \theta)$.

116.

Function	$\sin x$	$\cos x$	$\tan x$
Domain	$(-\infty, \infty)$	$(-\infty, \infty)$	All reals except $\frac{\pi}{2} + n\pi$
Range	$[-1, 1]$	$[-1, 1]$	$(-\infty, \infty)$
Evenness	No	Yes	No
Oddness	Yes	No	Yes
Period	2π	2π	π
Zeros	$n\pi$	$\frac{\pi}{2} + n\pi$	$n\pi$

Function	$\csc x$	$\sec x$	$\cot x$
Domain	All reals except $n\pi$	All reals except $\frac{\pi}{2} + n\pi$	All reals except $n\pi$
Range	$(-\infty, -1] \cup [1, \infty)$	$(-\infty, -1] \cup [1, \infty)$	$(-\infty, \infty)$
Evenness	No	Yes	No
Oddness	Yes	No	Yes
Period	2π	2π	π
Zeros	None	None	$\frac{\pi}{2} + n\pi$

Patterns and conclusions may vary.

117. $3x - 7 = 14$

$3x = 21$

$x = 7$

118. $44 - 9x = 61$

$9x = -17$

$x = -\frac{17}{9} \approx -1.889$

119. $x^2 - 2x - 5 = 0$

$$x = \frac{2 \pm \sqrt{4 + 20}}{2} = 1 \pm \sqrt{6}$$

$x \approx 3.449, x \approx -1.449$

120. $2x^2 + x - 4 = 0$

$$x = \frac{-1 \pm \sqrt{1 + 4(4)(2)}}{4} = \frac{-1 \pm \sqrt{33}}{4}$$

$x \approx 1.186, -1.686$

121. $\dfrac{3}{x-1} = \dfrac{x+2}{9}$

$27 = (x-1)(x+2)$

$x^2 + x - 29 = 0$

$x = \dfrac{-1 \pm \sqrt{1 + 4(29)}}{2} = \dfrac{-1 \pm \sqrt{117}}{2}$

$x \approx -5.908, 4.908$

122. $\dfrac{5}{x} = \dfrac{x+4}{2x}$

$10x = x^2 + 4x$

$x^2 - 6x = 0$

$x(x-6) = 0$

$x = 6 \quad (x = 0 \text{ extraneous})$

123. $4^{3-x} = 726$

$3 - x = \log_4 726$

$x = 3 - \log_4 726 = 3 - \dfrac{\ln 726}{\ln 4} \approx -1.752$

124. $\dfrac{4500}{4 + e^{2x}} = 50$

$90 = 4 + e^{2x}$

$86 = e^{2x}$

$2x = \ln 86$

$x = \dfrac{1}{2} \ln 86 \approx 2.227$

125. $\ln x = -6$

$x = e^{-6} \approx 0.002479 \approx 0.002$

126. $\ln \sqrt{x+10} = \frac{1}{2} \ln(x+10) = 1 \implies \ln(x+10) = 2 \implies x + 10 = e^2 \implies x = e^2 - 10 \approx -2.611$

Section 4.5 Graphs of Sine and Cosine Functions

- You should be able to graph $y = a \sin(bx - c)$ and $y = a \cos(bx - c)$.
- Amplitude: $|a|$
- Period: $\dfrac{2\pi}{|b|}$
- Shift: Solve $bx - c = 0$ and $bx - c = 2\pi$.
- Key increments: $\dfrac{1}{4}$ (period)

Vocabulary Check

1. amplitude **2.** one cycle **3.** $\dfrac{2\pi}{b}$ **4.** phase shift

1. $f(x) = \sin x$

(a) x-intercepts: $(-2\pi, 0), (-\pi, 0), (0, 0), (\pi, 0), (2\pi, 0)$

(b) y-intercept: $(0, 0)$

(c) Increasing on: $\left(-2\pi, -\frac{3\pi}{2}\right), \left(-\frac{\pi}{2}, \frac{\pi}{2}\right), \left(\frac{3\pi}{2}, 2\pi\right)$

Decreasing on: $\left(-\frac{3\pi}{2}, -\frac{\pi}{2}\right), \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

(d) Relative maxima: $\left(-\frac{3\pi}{2}, 1\right), \left(\frac{\pi}{2}, 1\right)$

Relative minima: $\left(-\frac{\pi}{2}, -1\right), \left(\frac{3\pi}{2}, -1\right)$

2. $f(x) = \cos x$

(a) x-intercepts: $\left(-\frac{3\pi}{2}, 0\right), \left(-\frac{\pi}{2}, 0\right), \left(\frac{\pi}{2}, 0\right), \left(\frac{3\pi}{2}, 0\right)$

(b) y-intercept: $(0, 1)$

(c) Increasing on: $(-\pi, 0), (\pi, 2\pi)$

Decreasing on: $(-2\pi, -\pi), (0, \pi)$

(d) Relative maxima: $(-2\pi, 1), (0, 1), (2\pi, 1)$

Relative minima: $(-\pi, -1), (\pi, -1)$

3. $y = 3 \sin 2x$

Period: $\frac{2\pi}{2} = \pi$

Amplitude: $|3| = 3$

Xmin = -2π
Xmax = 2π
Xscl = $\pi/2$
Ymin = -4
Ymax = 4
Yscl = 1

4. $y = 2 \cos 3x$

Period: $\frac{2\pi}{b} = \frac{2\pi}{3}$

Amplitude: $|a| = 2$

Xmin = $-\pi$
Xmax = π
Xscl = $\pi/4$
Ymin = -3
Ymax = 3
Yscl = 1

5. $y = \frac{5}{2} \cos \frac{x}{2}$

Period: $\frac{2\pi}{1/2} = 4\pi$

Amplitude: $\left|\frac{5}{2}\right| = \frac{5}{2}$

Xmin = -4π
Xmax = 4π
Xscl = π
Ymin = -3
Ymax = 3
Yscl = 1

6. $y = -3 \sin \frac{x}{3}$

Period: $\frac{2\pi}{b} = \frac{2\pi}{(1/3)} = 6\pi$

Amplitude: $|a| = |-3| = 3$

Xmin = -6π
Xmax = 6π
Xscl = π
Ymin = -4
Ymax = 4
Yscl = 1

7. $y = \frac{2}{3} \sin \pi x$

Period: $\frac{2\pi}{\pi} = 2$

Amplitude: $\left|\frac{2}{3}\right| = \frac{2}{3}$

8. $y = \frac{3}{2} \cos \frac{\pi x}{2}$

Period: $\frac{2\pi}{b} = \frac{2\pi}{(\pi/2)} = 4$

Amplitude: $|a| = \frac{3}{2}$

9. $y = -2 \sin x$

Period: $\dfrac{2\pi}{1} = 2\pi$

Amplitude: $|-2| = 2$

10. $y = -\cos \dfrac{2x}{5}$

Period: $\dfrac{2\pi}{b} = \dfrac{2\pi}{2/5} = 5\pi$

Amplitude: $|a| = |-1| = 1$

11. $y = \dfrac{1}{4} \cos \dfrac{2x}{3}$

Period: $\dfrac{2\pi}{2/3} = 3\pi$

Amplitude: $\left|\dfrac{1}{4}\right| = \dfrac{1}{4}$

12. $y = \dfrac{5}{2} \cos \dfrac{x}{4}$

Period: $\dfrac{2\pi}{b} = \dfrac{2\pi}{1/4} = 8\pi$

Amplitude: $|a| = \dfrac{5}{2}$

13. $y = \dfrac{1}{3} \sin 4\pi x$

Period: $\dfrac{2\pi}{4\pi} = \dfrac{1}{2}$

Amplitude: $\left|\dfrac{1}{3}\right| = \dfrac{1}{3}$

14. $y = \dfrac{3}{4} \cos \dfrac{\pi x}{12}$

Period: $\dfrac{2\pi}{b} = \dfrac{2\pi}{\pi/12} = 24$

Amplitude: $|a| = \dfrac{3}{4}$

15. $f(x) = \sin x$

$g(x) = \sin(x - \pi)$

The graph of g is a horizontal shift to the right π units of the graph of f (a phase shift).

16. $f(x) = \cos x$, $g(x) = \cos(x + \pi)$

g is a horizontal shift of f π units to the left.

17. $f(x) = \cos 2x$

$g(x) = -\cos 2x$

The graph of g is a reflection in the x-axis of the graph of f.

18. $f(x) = \sin 3x$, $g(x) = \sin(-3x)$

g is a reflection of f about the y-axis. (or, about the x-axis)

19. $f(x) = \cos x$

$g(x) = -5 \cos x$

The graph of g has five times the amplitude of f, and reflected in the x-axis.

20. $f(x) = \sin x$, $g(x) = -\frac{1}{2} \sin x$

The amplitude of g is one-half that of f. g is a reflection of f in the x-axis.

21. $f(x) = \sin 2x$

$g(x) = 5 + \sin 2x$

The graph of g is a vertical shift upward of five units of the graph of f.

22. $f(x) = \cos 4x$, $g(x) = -6 + \cos 4x$

g is a vertical shift of f six units downward.

23. The graph of g has twice the amplitude as the graph of f. The period is the same.

24. The period of g is one-half the period of f.

25. The graph of g is a horizontal shift π units to the right of the graph of f.

26. Shift the graph of f two units upward to obtain the graph of g.

27. $f(x) = \sin x$

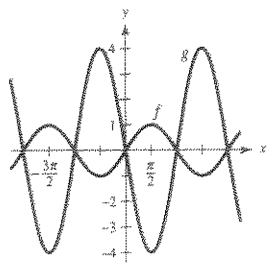
Period: 2π

Amplitude: 1

$g(x) = -4 \sin x$

Period: 2π

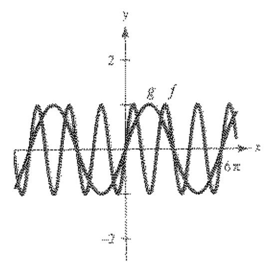
Amplitude: $|-4| = 4$



28. $f(x) = \sin x$

$g(x) = \sin \dfrac{x}{3}$

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin x$	0	1	0	-1	0
$\sin \frac{x}{3}$	0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$

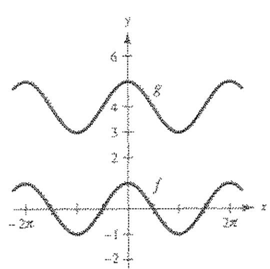


29. $f(x) = \cos x$

Period: 2π

Amplitude: 1

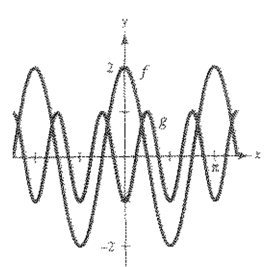
$g(x) = 4 + \cos x$ is a vertical shift of the graph of $f(x)$ four units upward.



30. $f(x) = 2 \cos 2x$

$g(x) = -\cos 4x$

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
$2 \cos 2x$	2	0	-2	0	2
$-\cos 4x$	-1	1	-1	1	-1

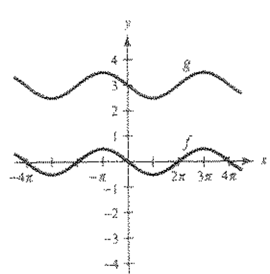


31. $f(x) = -\dfrac{1}{2} \sin \dfrac{x}{2}$

Period: 4π

Amplitude: $\dfrac{1}{2}$

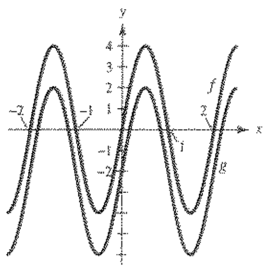
$g(x) = 3 - \dfrac{1}{2} \sin \dfrac{x}{2}$ is the graph of $f(x)$ shifted vertically three units upward.



32. $f(x) = 4 \sin \pi x$

$g(x) = 4 \sin \pi x - 2$

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$f(x)$	0	4	0	-4	0
$g(x)$	-2	2	-2	-6	-2

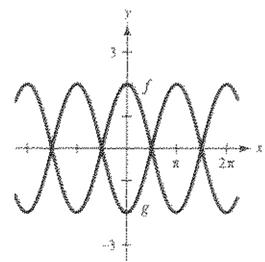


33. $f(x) = 2 \cos x$

Period: 2π

Amplitude: 2

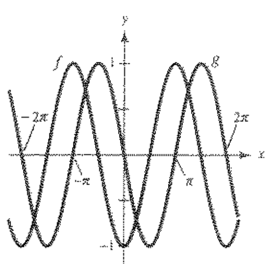
$g(x) = 2 \cos(x + \pi)$ is the graph of $f(x)$ shifted π units to the left.



34. $f(x) = -\cos x$

$g(x) = -\cos\left(x - \frac{\pi}{2}\right)$

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$-\cos x$	-1	0	1	0	-1
$-\cos\left(x - \frac{\pi}{2}\right)$	0	-1	0	1	0

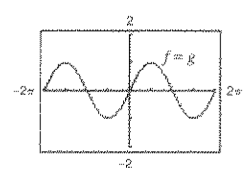


35. $f(x) = \sin x,\ g(x) = \cos\left(x - \frac{\pi}{2}\right)$

$\sin x = \cos\left(x - \frac{\pi}{2}\right)$

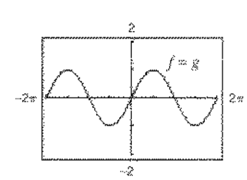
Period: 2π

Amplitude: 1



36. $f(x) = \sin x,\ g(x) = -\cos\left(x + \frac{\pi}{2}\right)$

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin x$	0	1	0	-1	0
$-\cos\left(x + \frac{\pi}{2}\right)$	0	1	0	-1	0

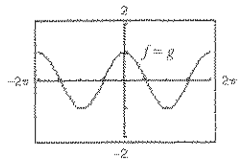


Conjecture: $\sin x = -\cos\left(x + \frac{\pi}{2}\right)$

37. $f(x) = \cos x$

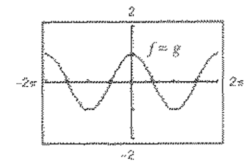
$g(x) = -\sin\left(x - \frac{\pi}{2}\right) = \sin\left(\frac{\pi}{2} - x\right) = \cos x$

Thus, $f(x) = g(x)$.



38. $f(x) = \cos x$, $g(x) = -\cos(x - \pi)$

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\cos x$	1	0	-1	0	1
$-\cos(x - \pi)$	1	0	-1	0	1



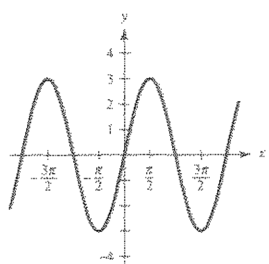
Conjecture: $\cos x = -\cos(x - \pi)$

39. $y = 3 \sin x$

Period: 2π

Amplitude: 3

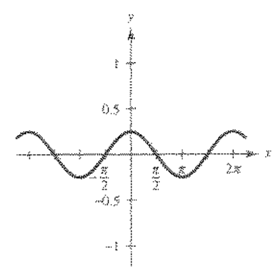
Key points: $(0, 0)$, $\left(\frac{\pi}{2}, 3\right)$, $(\pi, 0)$, $\left(\frac{3\pi}{2}, -3\right)$, $(2\pi, 0)$



40. $y = \frac{1}{4} \cos x$

Period: 2π

Amplitude: $\frac{1}{4}$

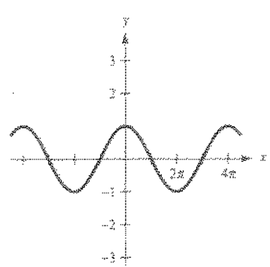


41. $y = \cos \frac{x}{2}$

Period: 4π

Amplitude: 1

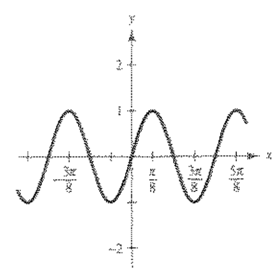
Key points: $(0, 1)$, $(\pi, 0)$, $(2\pi, -1)$, $(3\pi, 0)$, $(4\pi, 1)$



42. $y = \sin 4x$

Period: $\frac{2\pi}{4} = \frac{\pi}{2}$

Amplitude: 1



43. $y = \sin\left(x - \frac{\pi}{4}\right)$; $a = 1,\ b = 1,\ c = \frac{\pi}{4}$

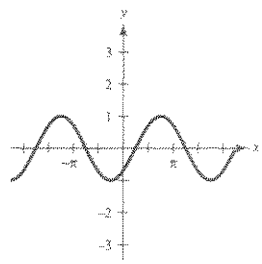
Period: 2π

Amplitude: 1

Shift: Set $x - \frac{\pi}{4} = 0$ and $x - \frac{\pi}{4} = 2\pi$

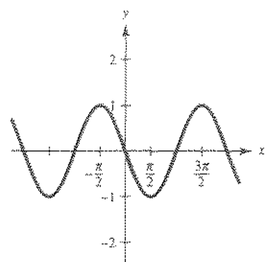
$$x = \frac{\pi}{4} \qquad x = \frac{9\pi}{4}$$

Key points: $\left(\frac{\pi}{4}, 0\right), \left(\frac{3\pi}{4}, 1\right), \left(\frac{5\pi}{4}, 0\right), \left(\frac{7\pi}{4}, -1\right), \left(\frac{9\pi}{4}, 0\right)$



44. $y = \sin(x - \pi)$

Horizontal shift π units to the right

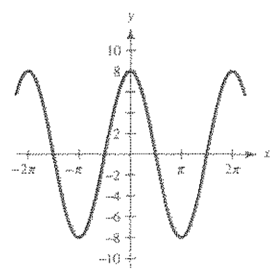


45. $y = -8\cos(x + \pi)$

Period: 2π

Amplitude: 8

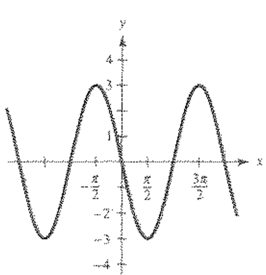
Key points: $(-\pi, -8), \left(-\frac{\pi}{2}, 0\right), (0, 8), \left(\frac{\pi}{2}, 0\right), (\pi, -8)$



46. $y = 3\cos\left(x + \frac{\pi}{2}\right)$

Amplitude: 3

Horizontal shift $\frac{\pi}{2}$ units to the left

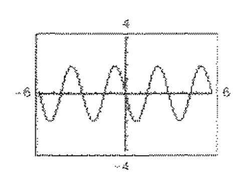


Note: $3\cos\left(x + \frac{\pi}{2}\right) = -3\sin x$

47. $y = -2\sin\frac{2\pi x}{3}$

Amplitude: 2

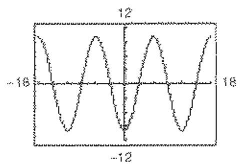
Period: $\frac{2\pi}{2\pi/3} = 3$



48. $y = -10 \cos \dfrac{\pi x}{6}$

Amplitude: 10

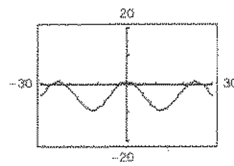
Period: $\dfrac{2\pi}{\pi/6} = 12$



49. $y = -4 + 5 \cos \dfrac{\pi t}{12}$

Amplitude: 5

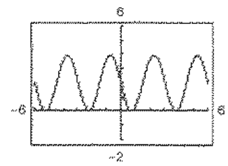
Period: $\dfrac{2\pi}{\pi/12} = 24$



50. $y = 2 - 2 \sin \dfrac{2\pi x}{3}$

Amplitude: 2

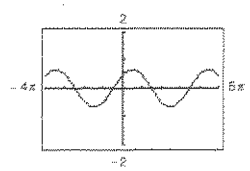
Period: $\dfrac{2\pi}{2\pi/3} = 3$



51. $y = \dfrac{2}{3} \cos\left(\dfrac{x}{2} - \dfrac{\pi}{4}\right)$

Amplitude: $\dfrac{2}{3}$

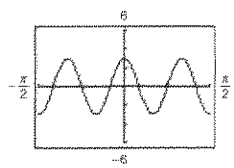
Period: $\dfrac{2\pi}{1/2} = 4\pi$



52. $y = -3 \cos(6x + \pi)$

Amplitude: 3

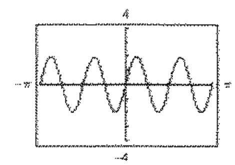
Period: $\dfrac{2\pi}{6} = \dfrac{\pi}{3}$



53. $y = -2 \sin(4x + \pi)$

Amplitude: 2

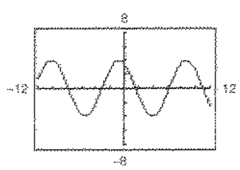
Period: $\dfrac{\pi}{2}$



54. $y = -4 \sin\left(\dfrac{2}{3}x - \dfrac{\pi}{3}\right)$

Amplitude: 4

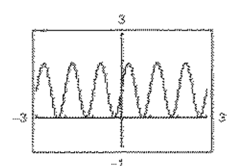
Period: 3π



55. $y = \cos\left(2\pi x - \dfrac{\pi}{2}\right) + 1$

Amplitude: 1

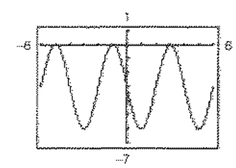
Period: 1



56. $y = 3 \cos\left(\dfrac{\pi x}{2} + \dfrac{\pi}{2}\right) - 3$

Amplitude: 3

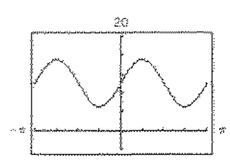
Period: 4



57. $y = 5 \sin(\pi - 2x) + 10$

Amplitude: 5

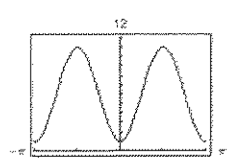
Period: π



58. $y = 5 \cos(\pi - 2x) + 6$

Amplitude: 5

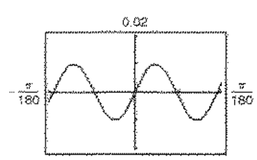
Period: π



59. $y = \frac{1}{100} \sin 120\pi t$

Amplitude: $\frac{1}{100}$

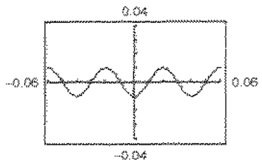
Period: $\frac{1}{60}$



60. $y = \frac{-1}{100}\cos(50\pi t)$

Amplitude: $\frac{1}{100}$

Period: $\frac{1}{25}$



61. $f(x) = a\cos x + d$

Amplitude: $\frac{1}{2}[8 - 0] = 4$

Since $f(x)$ is the graph of $g(x) = 4\cos x$ reflected about the x-axis and shifted vertically four units upward, we have $a = -4$ and $d = 4$. Thus,

$$f(x) = -4\cos x + 4$$
$$= 4 - 4\cos x.$$

62. $f(x) = a\cos x + d$

Amplitude: $\frac{-2 - (-4)}{2} = 1$

Reflected in the x-axis:

$a = -1$

$-4 = -1\cos 0 + d$

$d = -3$

$y = -3 - \cos x$

63. $f(x) = a\cos x + d$

Amplitude: $\frac{1}{2}[7 - (-5)] = 6$

Graph of f is the graph of $g(x) = 6\cos x$ reflected about the x-axis and shifted vertically one unit upward. Thus, $f(x) = -6\cos x + 1$.

64. $y = a\cos x + d$

Amplitude: $\frac{1}{2}$

Period: 2π

Reflected in x-axis, $a = -\frac{1}{2}$

$d = -4$

$y = -4 - \frac{1}{2}\cos x$

65. $f(x) = a\sin(bx - c)$

Amplitude: $|a| = 3$

Since the graph is reflected about the x-axis, we have $a = -3$.

Period: $\frac{2\pi}{b} = \pi \implies b = 2$

Phase shift: $c = 0$

Thus, $f(x) = -3\sin 2x$.

66. $y = a\sin(bx - c)$

Amplitude: $2 \implies a = 2$

Period: 4π

$\frac{2\pi}{b} = 4\pi \implies b = \frac{1}{2}$

Phase shift: $c = 0$

$y = 2\sin\left(\frac{x}{2}\right)$

67. $f(x) = a\sin(bx - c)$

Amplitude: $a = 1$

Period: $2\pi \implies b = 1$

Phase shift: $bx - c = 0$ when $x = \frac{\pi}{4}$.

$$(1)\left(\frac{\pi}{4}\right) - c = 0 \implies c = \frac{\pi}{4}$$

Thus, $f(x) = \sin\left(x - \frac{\pi}{4}\right)$.

68. $y = a\sin(bx - c)$

Amplitude: $2 \implies a = 2$

Period: 4

$\frac{2\pi}{b} = 4 \implies b = \frac{\pi}{2}$

Phase shift: $\frac{c}{b} = -1 \implies c = -\frac{\pi}{2}$

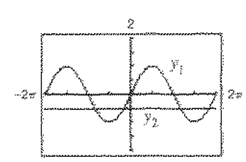
$y = 2\sin\left(\frac{\pi x}{2} + \frac{\pi}{2}\right)$

69. $y_1 = \sin x$

$y_2 = -\frac{1}{2}$

In the interval $[-2\pi, 2\pi]$, $\sin x = -\frac{1}{2}$ when

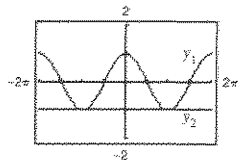
$$x = -\frac{5\pi}{6}, -\frac{\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}.$$



70. $y_1 = \cos x$

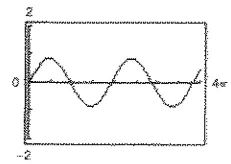
$y_2 = -1$

$y_1 = y_2$ when $x = \pi, -\pi$.



71. $v = 0.85 \sin \dfrac{\pi t}{3}$

(a)



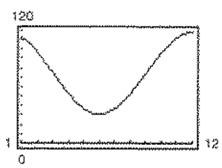
(b) Time for one cycle = one period = $\dfrac{2\pi}{\pi/3} = 6$ sec

(c) Cycles per min = $\dfrac{60}{6} = 10$ cycles per min

(d) The period would change.

72. $S = 74.50 + 43.75 \cos \dfrac{\pi t}{6}$

(a)

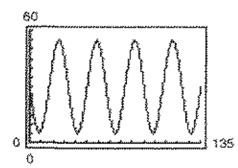


(b) Maximum sales: December ($t = 12$)

Minimum sales: June ($t = 6$)

73. $h = 25 \sin \dfrac{\pi}{15}(t - 75) + 30$

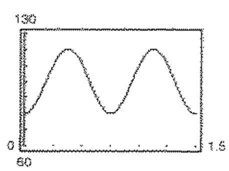
(a)



(b) Minimum: $30 - 25 = 5$ feet

Maximum: $30 + 25 = 55$ feet

74. $P = 100 - 20 \cos \dfrac{8\pi}{3} t$



Period: $\dfrac{2\pi}{(8\pi/3)} = \dfrac{3}{4}$

$\dfrac{1 \text{ heartbeat}}{(3/4)} \implies \dfrac{4}{3}$ heartbeats/second = 80 heartbeats/minute

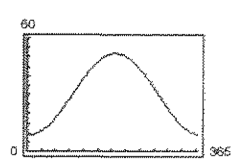
75. $C = 30.3 + 21.6 \sin\left(\dfrac{2\pi t}{365} + 10.9\right)$

(a) Period: $\dfrac{2\pi}{b} = \dfrac{2\pi}{(2\pi/365)} = 365$ days

This is to be expected: 365 days = 1 year

(b) The constant 30.3 gallons is the average daily fuel consumption.

(c)



Consumption exceeds 40 gallons/day when $124 \le x \le 252$. (Graph C together with $y = 40$.) (Beginning of May through part of September)

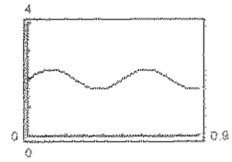
76. (a) Yes, y is a function of t because for each value of t there corresponds one and only one value of y.

(b) The period is approximately $2(0.375 - 0.125) = 0.5$ seconds.

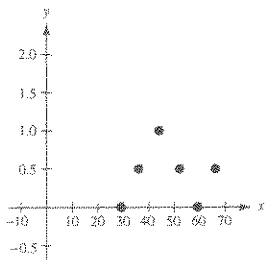
The amplitude is approximately $\frac{1}{2}(2.35 - 1.65) = 0.35$ centimeters.

(c) One model is $y = 0.35 \sin 4\pi t + 2$.

(d)

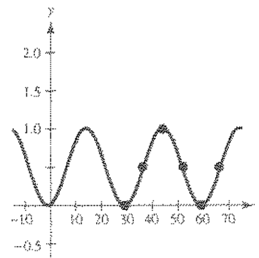


77. (a)



(b) $y = 0.506 \sin(0.209x - 1.336) + 0.526$

(c)



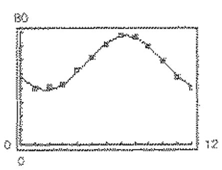
The model is a good fit.

(d) The period is $\dfrac{2\pi}{0.209} \approx 30.06$.

(e) June 29, 2007 is day 545. Using the model, $y \approx 0.2709$ or 27.09%.

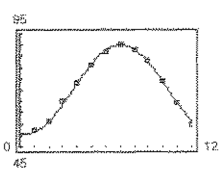
78. (a) $A(t) = 19.73 \sin(0.472t - 1.74) + 70.2$

(b)



The model somewhat fits the data.

(c)



The model is a good fit.

(d) Nantucket: 58°

Athens: 70.2°

The constant term (d) gives the average daily high temperature.

(e) Period for $N(t) = \dfrac{2\pi}{(2\pi/11)} = 11$

Period for $A(t) = \dfrac{2\pi}{0.472} \approx 13$

You would expect the period to be 12 (1 year).

(f) Athens has greater variability. This is given by the amplitude.

79. True. The period is $\dfrac{2\pi}{3/10} = \dfrac{20\pi}{3}$.

80. False. The amplitude is $\dfrac{1}{2}$ that of $y = \cos x$.

81. True

82. Answers will vary.

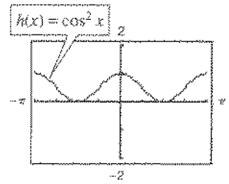
83. The graph passes through $(0, 0)$ and has period π. Matches (e).

84. The amplitude is 4 and the period 2π. Since $(0, -4)$ is on the graph, matches (a).

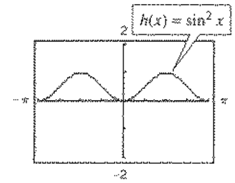
85. The period is 4π and the amplitude is 1. Since $(0, 1)$ and $(\pi, 0)$ are on the graph, matches (c).

86. The period is π. Since $(0, -1)$ is on the graph, matches (d).

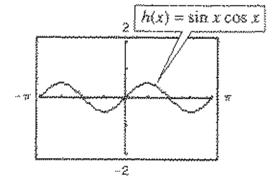
87. (a) $h(x) = \cos^2 x$ is even.



(b) $h(x) = \sin^2 x$ is even.



(c) $h(x) = \sin x \cos x$ is odd.



88. (a) In Exercise 87, $f(x) = \cos x$ is even and we saw that $h(x) = \cos^2 x$ is even. Therefore, for $f(x)$ even and $h(x) = [f(x)]^2$, we make the conjecture that $h(x)$ is even.

(b) In Exercise 87, $g(x) = \sin x$ is odd and we saw that $h(x) = \sin^2 x$ is even. Therefore, for $g(x)$ odd and $h(x) = [g(x)]^2$, we make the conjecture that $h(x)$ is even.

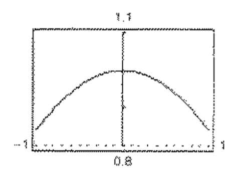
(c) From part (c) of 87, we conjecture that the product of an even function and an odd function is odd.

89. (a)

x	-1	-0.1	-0.01	-0.001
$\dfrac{\sin x}{x}$	0.8415	0.9983	1.0	1.0

x	0	0.001	0.01	0.1	1
$\dfrac{\sin x}{x}$	Undef.	1.0	1.0	0.9983	0.8415

(b)



As $x \to 0$, $f(x) = \dfrac{\sin x}{x}$ approaches 1.

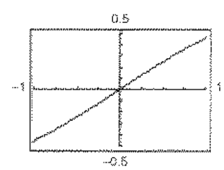
(c) As x approaches 0, $\dfrac{\sin x}{x}$ approaches 1.

90. (a)

x	-1	-0.1	-0.01	-0.001
$\dfrac{1 - \cos x}{x}$	-0.4597	-0.05	-0.005	-0.0005

x	0	0.001	0.01	0.1	1
$\dfrac{1 - \cos x}{x}$	Undef.	0.0005	0.005	0.05	0.4597

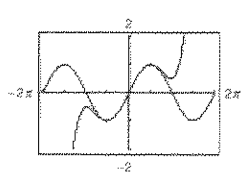
(b)



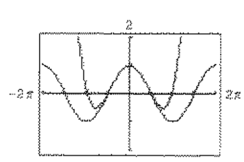
As $x \to 0$, $\dfrac{1 - \cos x}{x}$ approaches 0.

(c) As x approaches 0, $\dfrac{1 - \cos x}{x}$ approaches 0.

91. (a)

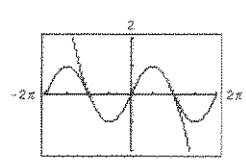


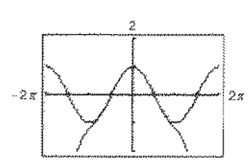
(b)



(c) Next term for sine approximation: $-\dfrac{x^7}{7!}$

Next term for cosine approximation: $-\dfrac{x^6}{6!}$





92. (a) $\sin(1) \approx 0.8415$

(b) $\sin\left(\frac{1}{2}\right) \approx 0.4794$

(c) $\sin\left(\frac{\pi}{8}\right) \approx 0.3827$

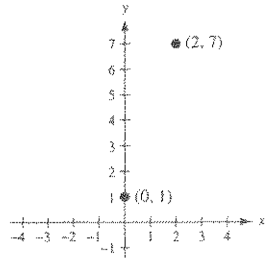
(d) $\cos(-1) \approx 0.5403$

(e) $\cos\left(-\frac{\pi}{4}\right) \approx 0.7071$

(f) $\cos\left(-\frac{1}{2}\right) \approx 0.8776$

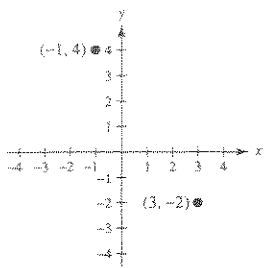
In all cases, the approximations are very accurate.

93.



$$\text{Slope} = \frac{7-1}{2-0} = 3$$

94.



$$m = \frac{-2-4}{3+1} = \frac{-6}{4} = \frac{-3}{2}$$

95. $8.5 = 8.5\left(\frac{180°}{\pi}\right) \approx 487.014°$

96. $-0.48 = -0.48\left(\frac{180°}{\pi}\right) \approx -27.502°$

97. Answers will vary. (Make a Decision)

Section 4.6 Graphs of Other Trigonometric Functions

- You should be able to graph:
  - $y = a\tan(bx - c)$ $\quad$ $y = a\cot(bx - c)$
  - $y = a\sec(bx - c)$ $\quad$ $y = a\csc(bx - c)$
- When graphing $y = a\sec(bx - c)$ or $y = a\csc(bx - c)$ you should know to first graph $y = a\cos(bx - c)$ or $y = a\sin(bx - c)$ since
  - (a) The intercepts of sine and cosine are vertical asymptotes of cosecant and secant.
  - (b) The maximums of sine and cosine are local minimums of cosecant and secant.
  - (c) The minimums of sine and cosine are local maximums of cosecant and secant.
- You should be able to graph using a damping factor.

Vocabulary Check

1. vertical $\quad$ **2.** reciprocal $\quad$ **3.** damping

1. $f(x) = \tan x$

(a) x-intercepts: $(-2\pi, 0), (-\pi, 0), (0, 0), (\pi, 0), (2\pi, 0)$

(b) y-intercept: $(0, 0)$

(c) Increasing on: $\left(-2\pi, -\frac{3\pi}{2}\right), \left(-\frac{3\pi}{2}, -\frac{\pi}{2}\right), \left(-\frac{\pi}{2}, \frac{\pi}{2}\right), \left(\frac{\pi}{2}, \frac{3\pi}{2}\right), \left(\frac{3\pi}{2}, 2\pi\right)$

Never decreasing

(d) No relative extrema

(e) Vertical asymptotes: $x = -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$

2. $f(x) = \cot x$

(a) x-intercepts: $\left(-\frac{3\pi}{2}, 0\right), \left(-\frac{\pi}{2}, 0\right), \left(\frac{\pi}{2}, 0\right), \left(\frac{3\pi}{2}, 0\right)$

(b) No y-intercepts

(c) Decreasing on: $(-2\pi, -\pi), (-\pi, 0), (0, \pi), (\pi, 2\pi)$

Never increasing

(d) No relative extrema

(e) Vertical asymptotes: $x = -2\pi, -\pi, 0, \pi, 2\pi$

3. $f(x) = \sec x$

(a) No x-intercepts

(b) y-intercept: $(0, 1)$

(c) Increasing on intervals:

$\left(-2\pi, -\frac{3\pi}{2}\right), \left(-\frac{3\pi}{2}, -\pi\right), \left(0, \frac{\pi}{2}\right), \left(\frac{\pi}{2}, \pi\right)$

Decreasing on intervals:

$\left(-\pi, -\frac{\pi}{2}\right), \left(-\frac{\pi}{2}, 0\right), \left(\pi, \frac{3\pi}{2}\right), \left(\frac{3\pi}{2}, 2\pi\right)$

(d) Relative minima: $(-2\pi, 1), (0, 1), (2\pi, 1)$,

Relative maxima: $(-\pi, -1), (\pi, -1)$

(e) Vertical asymptotes: $x = -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$

4. $f(x) = \csc x$

(a) No x-intercepts

(b) No y-intercepts

(c) Increasing on intervals:

$\left(-\frac{3\pi}{2}, -\pi\right), \left(-\pi, -\frac{\pi}{2}\right), \left(\frac{\pi}{2}, \pi\right), \left(\pi, \frac{3\pi}{2}\right)$

Decreasing on intervals:

$\left(-2\pi, -\frac{3\pi}{2}\right), \left(-\frac{\pi}{2}, 0\right), \left(0, \frac{\pi}{2}\right), \left(\frac{3\pi}{2}, 2\pi\right)$

(d) Relative minima: $\left(-\frac{3\pi}{2}, 1\right), \left(\frac{\pi}{2}, 1\right)$

Relative maxima: $\left(-\frac{\pi}{2}, -1\right), \left(\frac{3\pi}{2}, -1\right)$

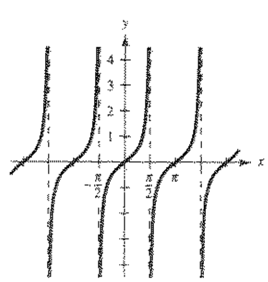
(e) Vertical asymptotes: $x = \pm\pi, \pm 2\pi$

5. $y = \frac{1}{2}\tan x$

Period: π

Two consecutive asymptotes: $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$

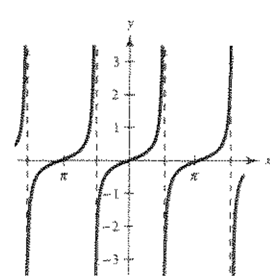
x	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$
y	$-\frac{1}{2}$	0	$\frac{1}{2}$



6. $y = \frac{1}{4}\tan x$

Period: π

Asymptotes: $x = \pm\frac{\pi}{2}$



7. $y = -2 \tan 2x$

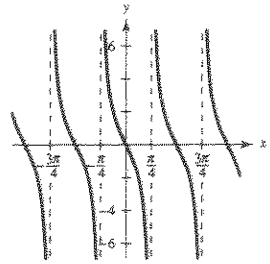
Period: $\frac{\pi}{2}$

Two consecutive asymptotes:

$2x = -\frac{\pi}{2} \implies x = -\frac{\pi}{4}$

$2x = \frac{\pi}{2} \implies x = \frac{\pi}{4}$

x	$-\frac{\pi}{8}$	0	$\frac{\pi}{8}$
y	2	0	-2



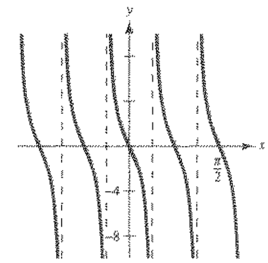
8. $y = -3 \tan 4x$

Period: $\frac{\pi}{4}$

Asymptotes:

$4x = \frac{-\pi}{2} \implies x = \frac{-\pi}{8}$

$4x = \frac{\pi}{2} \implies x = \frac{\pi}{8}$

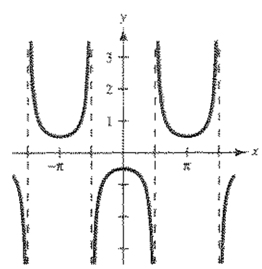


9. $y = -\frac{1}{2} \sec x$

Graph $y = -\frac{1}{2} \cos x$ first.

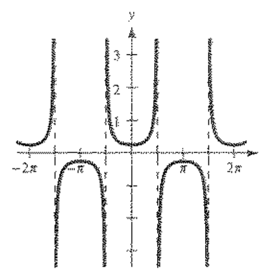
Period: 2π

One cycle: 0 to 2π



10. $y = \frac{1}{4} \sec x$

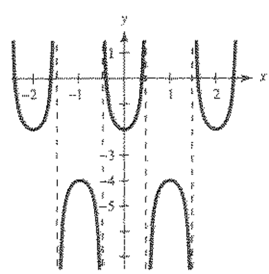
Period: 2π



11. $y = \sec \pi x - 3$

Period: 2

Shift graph of $\sec \pi x$ down three units.



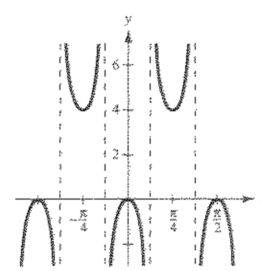
12. $y = -2 \sec 4x + 2$

Period: $\frac{2\pi}{4} = \frac{\pi}{2}$

Asymptotes:

$x = -\frac{\pi}{8}, x = \frac{\pi}{8}$

x	$-\frac{\pi}{16}$	0	$\frac{\pi}{16}$
y	-0.828	0	-0.828

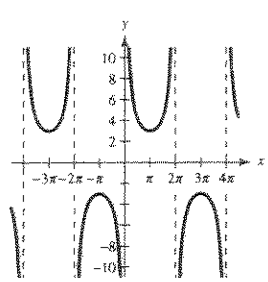


13. $y = 3 \csc \frac{x}{2}$

Graph $y = 3 \sin \frac{x}{2}$ first.

Period: $\frac{2\pi}{1/2} = 4\pi$

One cycle: 0 to 4π



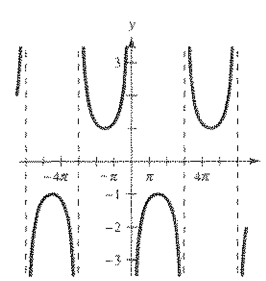
14. $y = -\csc \frac{x}{3}$

Period: $\frac{2\pi}{(1/3)} = 6\pi$

Asymptotes:

$x = 0, x = 3\pi$

x	π	2π	4π
y	-1.155	-1.155	1.155



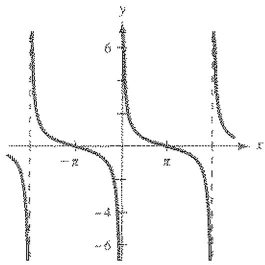
15. $y = \frac{1}{2}\cot\frac{x}{2}$

Period: $\frac{\pi}{1/2} = 2\pi$

Two consecutive asymptotes:

$\frac{x}{2} = 0 \implies x = 0$

$\frac{x}{2} = \pi \implies x = 2\pi$

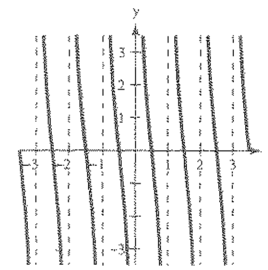


x	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
y	$\frac{1}{2}$	0	$-\frac{1}{2}$

16. $y = 3\cot\pi x$

Period: $\frac{\pi}{\pi} = 1$

Asymptotes: $x = 0, x = 1$



17. $y = 2\tan\frac{\pi x}{4}$

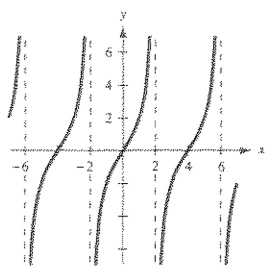
Period: $\frac{\pi}{\pi/4} = 4$

Two consecutive asymptotes:

$\frac{\pi x}{4} = -\frac{\pi}{2} \implies x = -2$

$\frac{\pi x}{4} = \frac{\pi}{2} \implies x = 2$

x	-1	0	1
y	-2	0	2

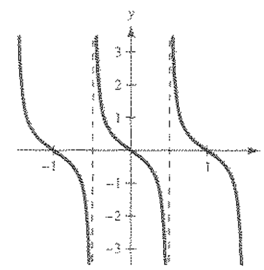


18. $y = -\frac{1}{2}\tan\pi x$

Period: 1

Asymptotes: $x = -\frac{1}{2}, x = \frac{1}{2}$

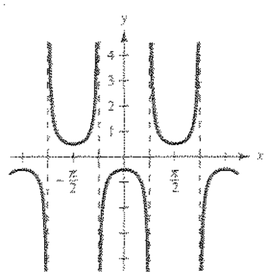
x	$-\frac{1}{4}$	0	$\frac{1}{4}$
y	$\frac{1}{2}$	0	$-\frac{1}{2}$



19. $y = \frac{1}{2}\sec(2x - \pi)$

Period: $\frac{2\pi}{2} = \pi$

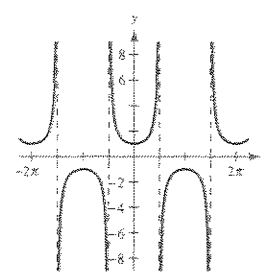
Asymptotes: $x = \pm\frac{\pi}{4}$



20. $y = -\sec(x + \pi)$

Period: 2π

Asymptotes: $x = \pm\frac{\pi}{2}$



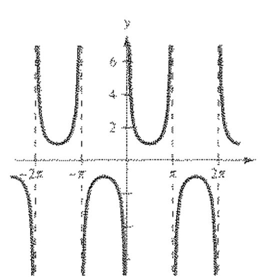
21. $y = \csc(\pi - x)$

Graph $y = \sin(\pi - x)$ first.

Period: 2π

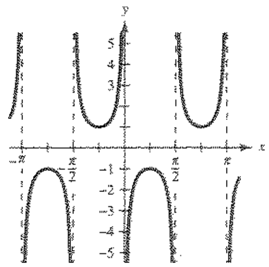
Asymptotes: Set $\pi - x = 0$ and $\pi - x = 2\pi$

$x = \pi \qquad x = -\pi$



22. $y = \csc(2x - \pi)$

Period: $\dfrac{2\pi}{2} = \pi$



23. $y = 2\cot\left(x - \dfrac{\pi}{2}\right)$

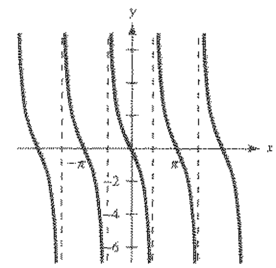
Period: π

Two consecutive asymptotes:

$x - \dfrac{\pi}{2} = 0 \implies x = \dfrac{\pi}{2}$

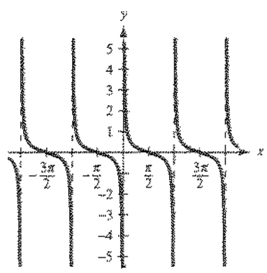
$x - \dfrac{\pi}{2} = \pi \implies x = \dfrac{3\pi}{2}$

x	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$
y	2	0	-2



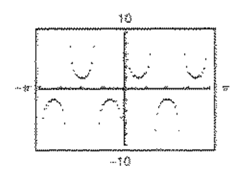
24. $y = \dfrac{1}{4}\cot(x + \pi)$

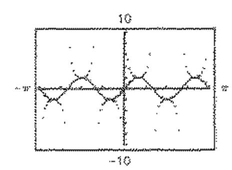
Period: π



25. $y = 2\csc 3x = \dfrac{2}{\sin(3x)}$

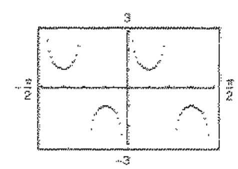
Period: $\dfrac{2\pi}{3}$

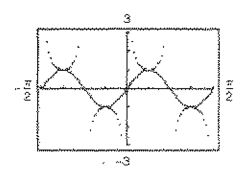




26. $y = -\csc(4x - \pi)$

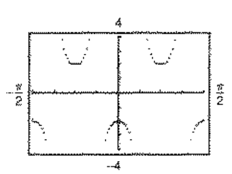
$y = \dfrac{-1}{\sin(4x - \pi)}$

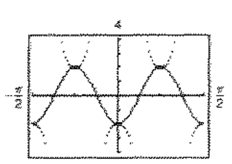




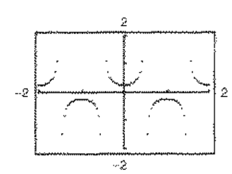
27. $y = -2\sec 4x$

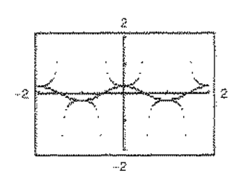
$= \dfrac{-2}{\cos 4x}$





28. $y = \dfrac{1}{4}\sec \pi x = \dfrac{1}{4\cos \pi x}$

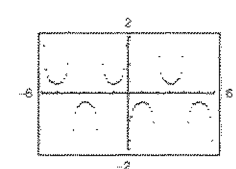


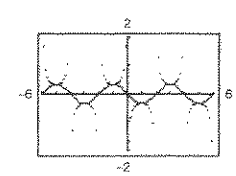


29. $y = \dfrac{1}{3}\sec\left(\dfrac{\pi x}{2} + \dfrac{\pi}{2}\right)$

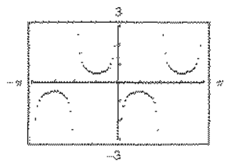
$= \dfrac{1}{3\cos\left(\dfrac{\pi x}{2} + \dfrac{\pi}{2}\right)}$

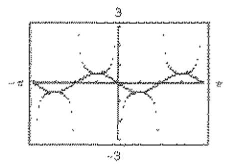
Period: 4





30. $y = \frac{1}{2}\csc(2x - \pi)$





31. $\tan x = 1$

$$x = -\frac{7\pi}{4}, -\frac{3\pi}{4}, \frac{\pi}{4}, \frac{5\pi}{4}$$

32. $\cot x = -\sqrt{3}$

The solutions appear to be:

$$x = -\frac{7\pi}{6}, -\frac{\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}$$

(or in decimal form: $-3.665, -0.524, 2.618, 5.760$)

33. $\sec x = -2$

$$x = \pm\frac{2\pi}{3}, \pm\frac{4\pi}{3}$$

34. $\csc x = \sqrt{2}$

The solutions appear to be:

$$-\frac{7\pi}{4}, -\frac{5\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}$$

(or in decimal form: $-5.498, -3.927, 0.785, 2.356$)

35. The graph of $f(x) = \sec x$ has y-axis symmetry. Thus, the function is even.

36. $f(x) = \tan x$

$\tan(-x) = -\tan x$

Thus, the function is odd and the graph of $y = \tan x$ is symmetric with the origin.

37. The function

$$f(x) = \csc 2x = \frac{1}{\sin 2x}$$

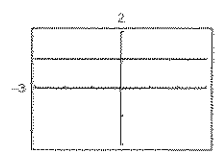
has origin symmetry. Thus, the function is odd.

38. $f(x) = \cot 2x$

$$f(-x) = \cot(-2x)$$
$$= \frac{\cos(-2x)}{\sin(-2x)} = -\frac{\cos(2x)}{\sin(2x)} = -f(x)$$

Odd

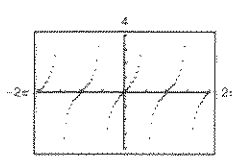
39. $y_1 = \sin x \csc x$ and $y_2 = 1$



Not equivalent because y_1 is not defined at 0.

$$\sin x \csc x = \sin x\left(\frac{1}{\sin x}\right) = 1, \quad \sin x \neq 0$$

40. $y_1 = \sin x \sec x$, $y_2 = \tan x$



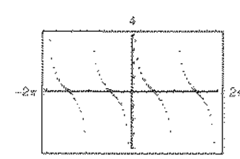
It appears that $y_1 = y_2$.

$$\sin x \sec x = \sin x \frac{1}{\cos x} = \frac{\sin x}{\cos x} = \tan x$$

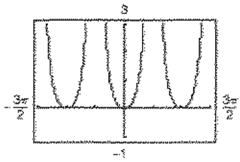
41. $y_1 = \frac{\cos x}{\sin x}$ and $y_2 = \cot x = \frac{1}{\tan x}$

Equivalent

$$\cot x = \frac{\cos x}{\sin x}$$



42. $y_1 = \sec^2 x - 1, y_2 = \tan^2 x$



It appears that $y_1 = y_2$.

$1 + \tan^2 x = \sec^2 x$

$\tan^2 x = \sec^2 x - 1$

43. $f(x) = x \cos x$

As $x \to 0, f(x) \to 0$.

Odd function

$f\left(\frac{3\pi}{2}\right) = 0$

Matches graph (d).

44. $f(x) = |x \sin x|$

Matches graph (a) as $x \to 0$, $f(x) \to 0$, and $f(x) \ge 0$ for all x.

45. $g(x) = |x| \sin x$

As $x \to 0, g(x) \to 0$.

Odd function

$g(2\pi) = 0$

Matches graph (b).

46. $g(x) = |x| \cos x$

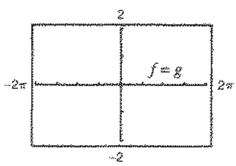
Even function

Matches graph (c) as $x \to 0, g(x) \to 0$.

47. $f(x) = \sin x + \cos\left(x + \frac{\pi}{2}\right), g(x) = 0$

$f(x) = g(x)$

The graph is the line $y = 0$.

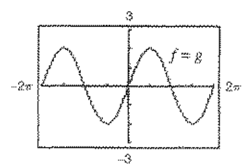


48. $f(x) = \sin x - \cos\left(x + \frac{\pi}{2}\right)$

$g(x) = 2 \sin x$

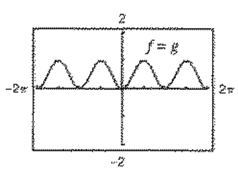
It appears that $f(x) = g(x)$. That is,

$\sin x - \cos\left(x + \frac{\pi}{2}\right) = 2 \sin x.$



49. $f(x) = \sin^2 x, g(x) = \frac{1}{2}(1 - \cos 2x)$

$f(x) = g(x)$

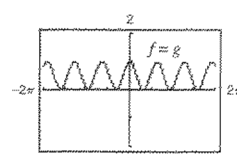


50. $f(x) = \cos^2 \frac{\pi x}{2}$

$g(x) = \frac{1}{2}(1 + \cos \pi x)$

It appears that $f(x) = g(x)$. That is, that

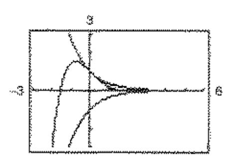
$\cos^2 \frac{\pi x}{2} = \frac{1}{2}(1 + \cos \pi x).$



51. $f(x) = e^{-x} \cos x$

Damping factor: e^{-x}

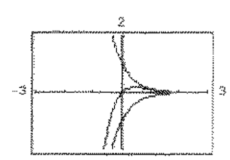
$x \to \infty, f(x) \to 0$



52. $f(x) = e^{-2x} \sin x$

Damping factor: e^{-2x}

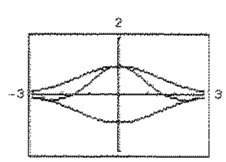
$f(x) \to 0$ as $x \to \infty$



53. $h(x) = e^{-x^2/4} \cos x$

Damping factor: $e^{-x^2/4}$

$h \to 0$ as $x \to \infty$

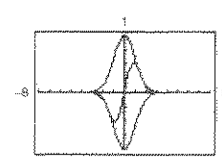


54. $g(x) = e^{-x^2/2} \sin x$

Damping factor: $y = e^{-x^2/2}$

$-e^{-x^2/2} \le g(x) \le e^{-x^2/2}$

$x \to \pm\infty, g(x) \to 0$



55. (a) As $x \to \frac{\pi^+}{2}, f(x) \to -\infty$.

(b) As $x \to \frac{\pi^-}{2}, f(x) \to \infty$.

(c) As $x \to -\frac{\pi^+}{2}, f(x) \to -\infty$.

(d) As $x \to -\frac{\pi^-}{2}, f(x) \to \infty$.

56. $f(x) = \sec x$

(a) As $x \to \frac{\pi^+}{2}, f(x) \to -\infty$.

(b) As $x \to \frac{\pi^-}{2}, f(x) \to \infty$.

(c) As $x \to -\frac{\pi^+}{2}, f(x) \to \infty$.

(d) As $x \to -\frac{\pi^-}{2}, f(x) \to -\infty$.

57. $f(x) = \cot x$

(a) As $x \to 0^+, f(x) \to \infty$.

(b) As $x \to 0^-, f(x) \to -\infty$.

(c) As $x \to \pi^+, f(x) \to \infty$.

(d) As $x \to \pi^-, f(x) \to -\infty$.

58. $f(x) = \csc x$

(a) As $x \to 0^+, f(x) \to \infty$.

(b) As $x \to 0^-, f(x) \to -\infty$.

(c) As $x \to \pi^+, f(x) \to -\infty$.

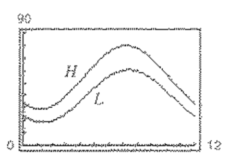
(d) As $x \to \pi^-, f(x) \to \infty$.

59. As the predator population increases, the number of prey decreases. When the number of prey is small, the number of predators decreases.

60. $H(t) = 54.33 - 20.38 \cos \frac{\pi t}{6} - 15.69 \sin \frac{\pi t}{6}$

$L(t) = 39.36 - 15.70 \cos \frac{\pi t}{6} - 14.16 \sin \frac{\pi t}{6}$

(a)



Period of $\cos \frac{\pi t}{6}$: $\frac{2\pi}{(\pi/6)} = 12$

Period of $\sin \frac{\pi t}{6}$: $\frac{2\pi}{(\pi/6)} = 12$

Period of $H(t)$: 12

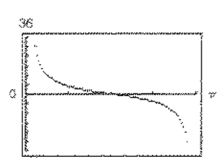
Period of $L(t)$: 12

(b) From the graph, it appears that the greatest difference between high and low temperatures occurs in summer. The smallest difference occurs in winter.

(c) The highest high and low temperatures appear to occur around the middle of July, roughly one month after the time when the sun is northernmost in the sky.

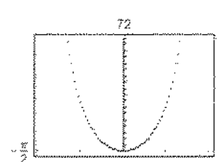
61. $\tan x = \frac{5}{d}$

$d = \frac{5}{\tan x} = 5 \cot x$

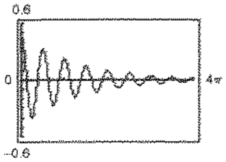


62. $\cos x = \frac{36}{d}$

$d = \frac{36}{\cos x} = 36 \sec x$



63. (a)



(b) The displacement function is not periodic, but damped. It approaches 0 as t increases.

64. (a) $\dfrac{1700}{2} = 850$ rev/min

(b) The direction of the saw is reversed.

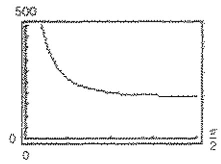
(c) $L = 60\left[\left(\dfrac{\pi}{2} + \phi\right) + \cot \phi\right],\ 0 < \phi < \dfrac{\pi}{2}$

(d)

ϕ	0.3	0.6	0.9	1.2	1.5
L	306.2	217.9	195.9	189.6	188.5

(e) Straight line lengths change faster.

(f)

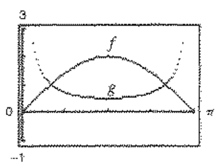


65. True. $-\dfrac{3\pi}{2} + \pi = -\dfrac{\pi}{2}$ and $x = -\dfrac{\pi}{2}$ is a vertical asymptote for the tangent function.

66. True. For $x \to -\infty$, $2^x \to 0$.

67. $f(x) = 2 \sin x$, $g(x) = \dfrac{1}{2} \csc x$

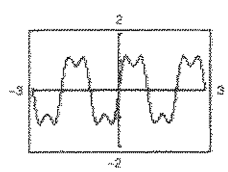
(a)



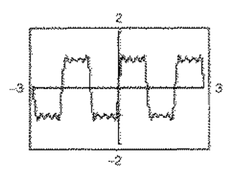
(b) $f(x) > g(x)$ for $\dfrac{\pi}{6} < x < \dfrac{5\pi}{6}$

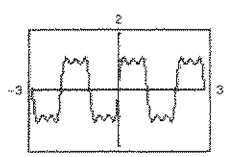
(c) As $x \to \pi$, $2 \sin x \to 0$ and $\dfrac{1}{2} \csc x \to \infty$, since $g(x)$ is the reciprocal of $f(x)$.

68. (a)



(b) $y_3 = \dfrac{4}{\pi}\left(\sin \pi x + \dfrac{1}{3} \sin 3\pi x + \dfrac{1}{5} \sin 5\pi x + \dfrac{1}{7} \sin 7\pi x\right)$





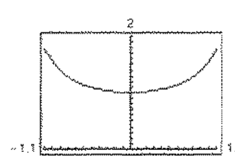
(c) $y_4 = y_3 + \dfrac{4}{\pi}\left(\dfrac{1}{9} \sin 9\pi x\right)$

69. (a)

x	-1	-0.1	-0.01	-0.001
$\dfrac{\tan x}{x}$	1.5574	1.0033	1.0	1.0

x	0	0.001	0.01	0.1	1
$\dfrac{\tan x}{x}$	Undef.	1.0	1.0	1.0033	1.5574

(b)



As $x \to 0$, $f(x) = \dfrac{\tan x}{x} \to 1$.

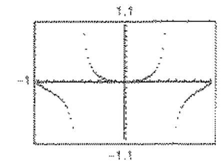
(c) The ratio approaches 1 as x approaches 0.

70. (a)

x	-1	-0.1	-0.01	-0.001
$\dfrac{\tan 3x}{3x}$	-0.0475	1.0311	1.0003	1.0

x	0	0.001	0.01	0.1	1
$\dfrac{\tan 3x}{3x}$	Undef.	1.0	1.0003	1.0311	-0.0475

(b)



As $x \to 0, f(x) = \dfrac{\tan 3x}{3x} \to 1.$

(c) The ratio approaches 1 as x approaches 0.

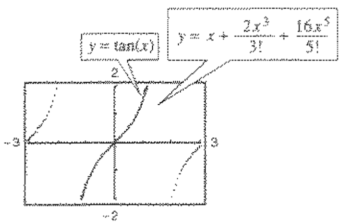
71. Period is $\frac{\pi}{2}$ and graph is increasing on $\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$.

Matches (a).

72. Period is $\frac{\pi}{2}$ and $\left(\frac{\pi}{8}, 1\right)$ is on the graph.

Matches (b).

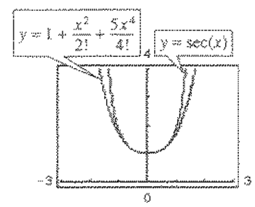
73.



The graphs of $y_1 = \tan x$ and $y_2 = x + \dfrac{2x^3}{3!} + \dfrac{16x^5}{5!}$

are similar on the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

74.



The graphs of $y_1 = \sec x$ and $y_2 = 1 + \dfrac{x^2}{2!} + \dfrac{5x^4}{4!}$

are similar on the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

75. Distributive Property

76. $7\left(\frac{1}{7}\right) = 1$

Multiplicative Inverse Property

77. Additive Identity Property

78. $(a + b) + 10 = a + (b + 10)$

Associative Property of Addition

79. Not one-to-one

80. Not one-to-one

81.
$$y = \sqrt{3x - 14},\ x \ge \tfrac{14}{3},\ y \ge 0$$
$$x = \sqrt{3y - 14},\ y \ge \tfrac{14}{3},\ x \ge 0$$
$$x^2 = 3y - 14$$
$$y = \tfrac{1}{3}(x^2 + 14)$$
$$f^{-1}(x) = \tfrac{1}{3}(x^2 + 14),\ x \ge 0$$

82. One-to-one
$$y = (x - 5)^{1/3}$$
$$x = (y - 5)^{1/3}$$
$$x^3 = y - 5$$
$$f^{-1}(x) = x^3 + 5$$

Section 4.7 Inverse Trigonometric Functions

- You should know the definitions, domains, and ranges of $y = \arcsin x$, $y = \arccos x$, and $y = \arctan x$.

Function	Domain	Range
$y = \arcsin x \implies x = \sin y$	$-1 \le x \le 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
$y = \arccos x \implies x = \cos y$	$-1 \le x \le 1$	$0 \le y \le \pi$
$y = \arctan x \implies x = \tan y$	$-\infty < x < \infty$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$

- You should know the inverse properties of the inverse trigonometric functions.

  $\sin(\arcsin x) = x,\ -1 \le x \le 1$ and $\arcsin(\sin y) = y,\ -\frac{\pi}{2} \le y \le \frac{\pi}{2}$

  $\cos(\arccos x) = x,\ -1 \le x \le 1$ and $\arccos(\cos y) = y,\ 0 \le y \le \pi$

  $\tan(\arctan x) = x$ and $\arctan(\tan y) = y,\ -\frac{\pi}{2} < y < \frac{\pi}{2}$

- You should be able to use the triangle technique to convert trigonometric functions or inverse trigonometric functions into algebraic expressions.

Vocabulary Check

1. $y = \sin^{-1} x,\ -1 \le x \le 1$

2. $y = \arccos x,\ 0 \le y \le \pi$

3. $y = \tan^{-1} x,\ -\infty < x < \infty,\ -\frac{\pi}{2} < y < \frac{\pi}{2}$

1. (a) $\arcsin \frac{1}{2} = \frac{\pi}{6}$ (b) $\arcsin 0 = 0$

2. (a) $y = \arccos \frac{1}{2} \implies \cos y = \frac{1}{2}$ for $0 \le y \le \pi \implies y = \frac{\pi}{3}$

(b) $y = \arccos 0 \implies \cos y = 0$ for $0 \le y \le \pi \implies y = \frac{\pi}{2}$

3. (a) $\arcsin 1 = \frac{\pi}{2}$ because $\sin \frac{\pi}{2} = 1$ and $-\frac{\pi}{2} \le \frac{\pi}{2} \le \frac{\pi}{2}$.

(b) $\arccos 1 = 0$ because $\cos 0 = 1$ and $0 \le 1 \le \pi$.

4. (a) $\arctan 1 = \frac{\pi}{4}$ because $\tan \frac{\pi}{4} = 1$ and $-\frac{\pi}{2} < \frac{\pi}{4} < \frac{\pi}{2}$.

(b) $\arctan 0 = 0$ because $\tan 0 = 0$ and $-\frac{\pi}{2} < 0 < \frac{\pi}{2}$.

5. (a) $\arctan \frac{\sqrt{3}}{3} = \frac{\pi}{6}$

(b) $\arctan(-1) = -\frac{\pi}{4}$

6. (a) $\arccos\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}$

(b) $\arcsin\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$

7. (a) $y = \arctan(-\sqrt{3}) \implies \tan y = -\sqrt{3}$ for $-\frac{\pi}{2} < y < \frac{\pi}{2} \implies y = -\frac{\pi}{3}$

(b) $y = \arctan \sqrt{3} \implies \tan y = \sqrt{3} \implies y = \frac{\pi}{3}$

8. (a) $y = \arccos\left(-\frac{1}{2}\right) \implies \cos y = -\frac{1}{2}$ for $0 \le y \le \pi \implies y = \frac{2\pi}{3}$

(b) $y = \arcsin \frac{\sqrt{2}}{2} \implies \sin y = \frac{\sqrt{2}}{2}$ for $-\frac{\pi}{2} \le y \le \frac{\pi}{2} \implies y = \frac{\pi}{4}$

9. (a) $y = \sin^{-1} \frac{\sqrt{3}}{2} \implies \sin y = \frac{\sqrt{3}}{2}$ for $-\frac{\pi}{2} \le y \le \frac{\pi}{2} \implies y = \frac{\pi}{3}$

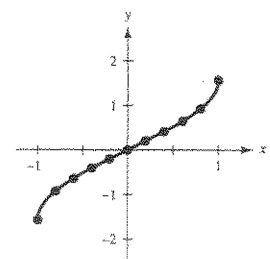
(b) $y = \tan^{-1}\left(\frac{-\sqrt{3}}{3}\right) \implies \tan y = \frac{-\sqrt{3}}{3} \implies y = -\frac{\pi}{6}$

10. (a)

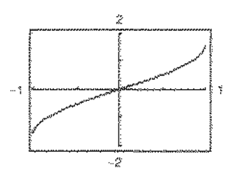
x	-1.0	-0.8	-0.6	-0.4	-0.2
y	-1.5708	-0.9273	-0.6435	-0.4115	-0.2014

x	0	0.2	0.4	0.6	0.8	1
y	0	0.2014	0.4115	0.6435	0.9273	1.5708

(b)



(c)



(d) (0, 0), symmetric to the origin

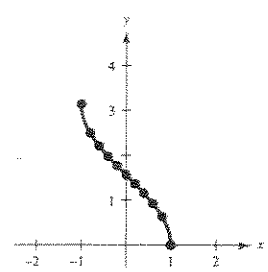
11. $y = \arccos x$

(a)

x	-1	-0.8	-0.6	-0.4	-0.2
y	3.1416	2.4981	2.2143	1.9823	1.7722

x	0	0.2	0.4	0.6	0.8	1.0
y	1.5708	1.3694	1.1593	0.9273	0.6435	0

(b)



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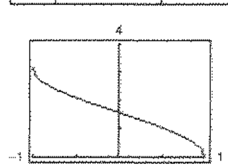


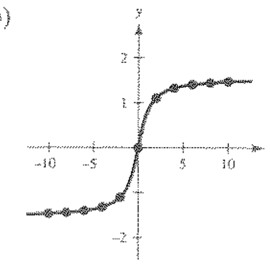
(d) Intercepts: $\left(0, \frac{\pi}{2}\right)$, (1, 0), no symmetry

12. (a)

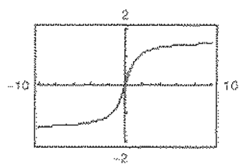
x	-10	-8	-6	-4	-2
y	-1.4711	-1.4464	-1.4056	-1.3258	-1.1071

x	0	2	4	6	8	10
y	0	1.1071	1.3258	1.4056	1.4464	1.4711

(b)



(c)



(d) Horizontal asymptotes: $y = \pm\dfrac{\pi}{2}$

13. $y = \arctan x \leftrightarrow \tan y = x$

$\left(-\sqrt{3}, -\dfrac{\pi}{3}\right), \left(-\dfrac{\sqrt{3}}{3}, -\dfrac{\pi}{6}\right), \left(1, \dfrac{\pi}{4}\right)$

14. $y = \arccos x$

$x = -1 \implies y = \pi, \quad (\cos \pi = -1)$

$x = -\dfrac{1}{2} \implies y = \dfrac{2\pi}{3}, \left(\cos \dfrac{2\pi}{3} = -\dfrac{1}{2}\right)$

$y = \dfrac{\pi}{6} \implies x = \dfrac{\sqrt{3}}{2}, \left(\cos \dfrac{\pi}{6} = \dfrac{\sqrt{3}}{2}\right)$

15. $\cos^{-1}(0.75) \approx 0.72$

16. $\sin^{-1} 0.56 \approx 0.59$

17. $\arcsin(-0.75) \approx -0.85$

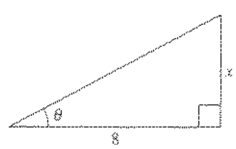
18. $\arccos(-0.7) \approx 2.35$

19. $\arctan(-6) \approx -1.41$

20. $\tan^{-1} 5.9 \approx 1.40$

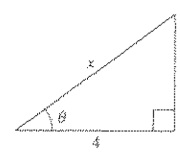
21. $\tan \theta = \dfrac{x}{8}$

$\theta = \arctan \dfrac{x}{8}$



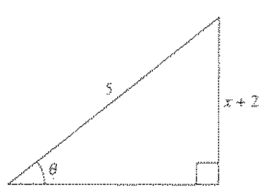
22. $\cos \theta = \dfrac{4}{x}$

$\theta = \arccos \dfrac{4}{x}$



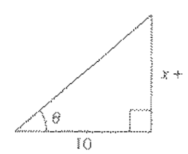
23. $\sin \theta = \dfrac{x+2}{5}$

$\theta = \arcsin\left(\dfrac{x+2}{5}\right)$



24. $\tan \theta = \dfrac{x+1}{10}$

$\theta = \arctan\left(\dfrac{x+1}{10}\right)$



25. Let y be the third side. Then

$y^2 = 2^2 - x^2 = 4 - x^2 \implies y = \sqrt{4 - x^2}$

$\sin \theta = \dfrac{x}{2} \implies \theta = \arcsin \dfrac{x}{2}$

$\cos \theta = \dfrac{\sqrt{4 - x^2}}{2} \implies \theta = \arccos \dfrac{\sqrt{4 - x^2}}{2}$

$\tan \theta = \dfrac{x}{\sqrt{4 - x^2}} \implies \theta = \arctan \dfrac{x}{\sqrt{4 - x^2}}$

26. Let y be the third side. Then

$y^2 = 3^2 - x^2 = 9 - x^2 \implies y = \sqrt{9 - x^2}.$

$\sin \theta = \dfrac{x}{3} \implies \theta = \arcsin \dfrac{x}{3}$

$\cos \theta = \dfrac{\sqrt{9 - x^2}}{3} \implies \theta = \arccos \dfrac{\sqrt{9 - x^2}}{3}$

$\tan \theta = \dfrac{x}{\sqrt{9 - x^2}} \implies \theta = \arctan \dfrac{x}{\sqrt{9 - x^2}}$

27. Let y be the hypotenuse. Then $y^2 = (x+1)^2 + 2^2 = x^2 + 2x + 5 \implies y = \sqrt{x^2+2x+5}$.

$$\sin\theta = \frac{x+1}{\sqrt{x^2+2x+5}} \implies \theta = \arcsin\frac{x+1}{\sqrt{x^2+2x+5}}$$

$$\cos\theta = \frac{2}{\sqrt{x^2+2x+5}} \implies \theta = \arccos\frac{2}{\sqrt{x^2+2x+5}}$$

$$\tan\theta = \frac{x+1}{2} \implies \theta = \arctan\frac{x+1}{2}$$

28. Let y be the hypotenuse. Then $y^2 = (x+2)^2 + 3^2 = x^2 + 4x + 13 \implies y = \sqrt{x^2+4x+13}$.

$$\sin\theta = \frac{x+2}{\sqrt{x^2+4x+13}} \implies \theta = \arcsin\frac{x+2}{\sqrt{x^2+4x+13}}$$

$$\cos\theta = \frac{3}{\sqrt{x^2+4x+13}} \implies \theta = \arccos\frac{3}{\sqrt{x^2+4x+13}}$$

$$\tan\theta = \frac{x+2}{3} \implies \theta = \arctan\frac{x+2}{3}$$

29. $\sin(\arcsin 0.7) = 0.7$

30. $\tan(\arctan 35) = 35$

31. $\cos[\arccos(-0.3)] = -0.3$

32. $\sin(\arcsin(-0.1)) = -0.1$

33. $\arcsin(\sin 3\pi) = \arcsin(0) = 0$

Note: 3π is not in the range of the arcsine function.

34. $\arccos\left(\cos\frac{7\pi}{2}\right) = \arccos(0) = \frac{\pi}{2}$

35. $\arctan\left(\tan\frac{11\pi}{6}\right) = \arctan\left(-\frac{\sqrt{3}}{3}\right) = -\frac{\pi}{6}$

36. $\arcsin\left(\sin\frac{7\pi}{4}\right) = \arcsin\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$

37. $\sin^{-1}\left(\sin\frac{5\pi}{2}\right) = \sin^{-1} 1 = \frac{\pi}{2}$

38. $\cos^{-1}\left(\cos\frac{3\pi}{2}\right) = \cos^{-1} 0 = \frac{\pi}{2}$

39. $\sin^{-1}\left(\tan\frac{5\pi}{4}\right) = \sin^{-1} 1 = \frac{\pi}{2}$

40. $\cos^{-1}\left(\tan\frac{3\pi}{4}\right) = \cos^{-1}(-1) = \pi$

41. $\tan(\arcsin 0) = \tan 0 = 0$

42. $\cos(\arctan(-1)) = \cos\left(-\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

43. $\sin(\arctan 1) = \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

44. $\sin(\arctan(-1)) = \sin\left(-\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

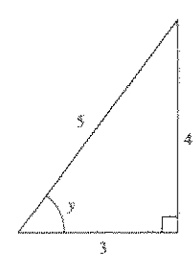
45. $\arcsin\left[\cos\left(-\frac{\pi}{6}\right)\right] = \arcsin\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$

46. $\arccos\left[\sin\left(-\frac{\pi}{6}\right)\right] = \arccos\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$

47. Let $y = \arctan\frac{4}{3}$. Then

$\tan y = \frac{4}{3}, 0 < y < \frac{\pi}{2}$, and

$\sin y = \frac{4}{5}$.

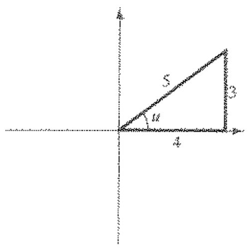


48. Let $u = \arcsin\frac{3}{5}$,

$\sin u = \frac{3}{5}, 0 < u < \frac{\pi}{2}$.

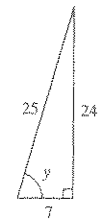
$\sec\left(\arcsin\frac{3}{5}\right) = \sec u$

$= \frac{\text{hyp}}{\text{adj}} = \frac{5}{4}$



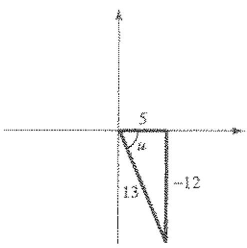
49. Let $y = \arcsin\frac{24}{25}$. Then

$\sin y = \frac{24}{25}$, and $\cos y = \frac{7}{25}$.



50. Let $u = \arctan\left(-\frac{12}{5}\right)$,

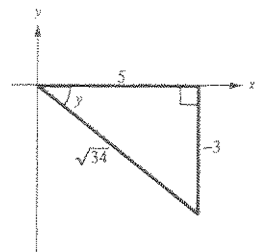
$\tan u = -\frac{12}{5}, -\frac{\pi}{2} < u < 0$.



$\csc\left[\arctan\left(-\frac{12}{5}\right)\right] = \csc u = \frac{\text{hyp}}{\text{opp}} = -\frac{13}{12}$

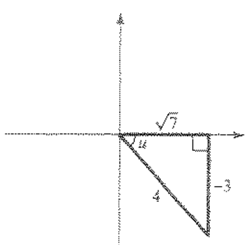
51. Let $y = \arctan\left(-\frac{3}{5}\right)$. Then,

$\tan y = -\frac{3}{5}, -\frac{\pi}{2} < y < 0$ and $\sec y = \frac{\sqrt{34}}{5}$.



52. Let $u = \arcsin\left(-\frac{3}{4}\right)$,

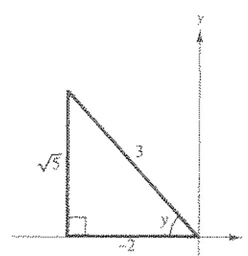
$\sin u = -\frac{3}{4}, -\frac{\pi}{2} < u < 0$.



$\tan\left(\arcsin\left(-\frac{3}{4}\right)\right) = \tan u = \frac{-3}{\sqrt{7}} = \frac{-3\sqrt{7}}{7}$

53. Let $y = \arccos\left(-\frac{2}{3}\right)$. Then,

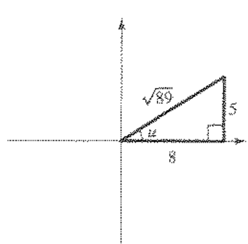
$\cos y = -\frac{2}{3}, \frac{\pi}{2} < y < \pi$ and $\sin y = \frac{\sqrt{5}}{3}$.



54. Let $u = \arctan\frac{5}{8}$,

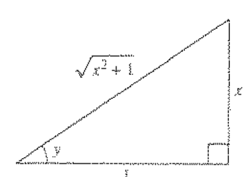
$\tan u = \frac{5}{8}, 0 < u < \frac{\pi}{2}$.

$\cot\left(\arctan\frac{5}{8}\right) = \cot u = \frac{8}{5}$

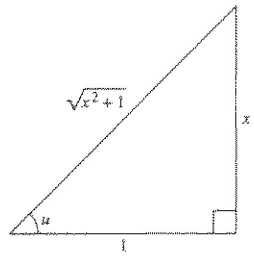


55. Let $y = \arctan x$. Then,

$\tan y = x$ and $\cot y = \frac{1}{x}$.



56. Let $u = \arctan x$, $\tan u = x = \dfrac{x}{1}$.

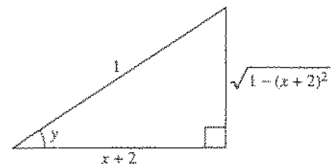


$\sin(\arctan x) = \sin u = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{x}{\sqrt{x^2 + 1}}$

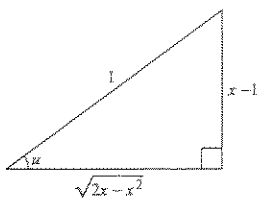
57. Let $y = \arccos(x + 2)$, $\cos y = x + 2$.

Opposite side: $\sqrt{1 - (x + 2)^2}$

$\sin y = \dfrac{\sqrt{1 - (x + 2)^2}}{1} = \sqrt{-x^2 - 4x - 3}$



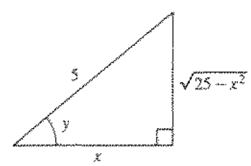
58. Let $u = \arcsin(x - 1)$, $\sin u = x - 1 = \dfrac{x - 1}{1}$.



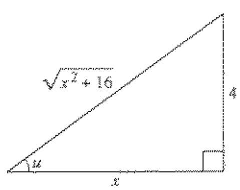
$\sec[\arcsin (x - 1)] = \sec u = \dfrac{\text{hyp}}{\text{adj}} = \dfrac{1}{\sqrt{2x - x^2}}$

59. Let $y = \arccos \dfrac{x}{5}$. Then $\cos y = \dfrac{x}{5}$, and

$\tan y = \dfrac{\sqrt{25 - x^2}}{x}$.



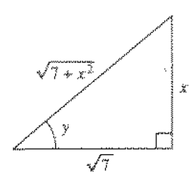
60. Let $u = \arctan \dfrac{4}{x}$, $\tan u = \dfrac{4}{x}$.



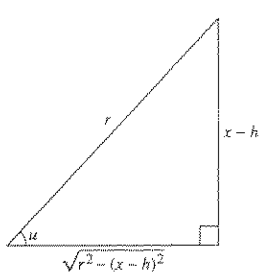
$\cot\left(\arctan \dfrac{4}{x}\right) = \cot u = \dfrac{\text{adj}}{\text{opp}} = \dfrac{x}{4}$

61. Let $y = \arctan \dfrac{x}{\sqrt{7}}$. Then $\tan y = \dfrac{x}{\sqrt{7}}$ and

$\csc y = \dfrac{\sqrt{7 + x^2}}{x}$.



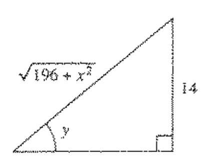
62. Let $u = \arcsin \dfrac{x - h}{r}$, $\sin u = \dfrac{x - h}{r}$.



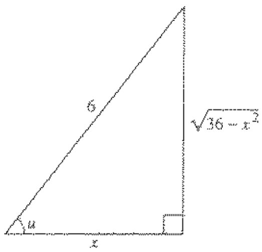
$\cos\left(\arcsin \dfrac{x - h}{r}\right) = \cos u = \dfrac{\sqrt{r^2 - (x - h)^2}}{r}$

63. Let $y = \arctan \dfrac{14}{x}$. Then $\tan y = \dfrac{14}{x}$ and

$\sin y = \dfrac{14}{\sqrt{196 + x^2}}$. Thus, $y = \arcsin\left(\dfrac{14}{\sqrt{196 + x^2}}\right)$.



64. If $\arcsin \dfrac{\sqrt{36 - x^2}}{6} = u$, then $\sin u = \dfrac{\sqrt{36 - x^2}}{6}$.



$$\arcsin \frac{\sqrt{36 - x^2}}{6} = \arccos \frac{x}{6}$$

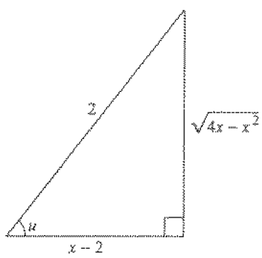
65. Let $y = \arccos \dfrac{3}{\sqrt{x^2 - 2x + 10}}$. Then,

$$\cos y = \frac{3}{\sqrt{x^2 - 2x + 10}} = \frac{3}{\sqrt{(x - 1)^2 + 9}}$$

and $\sin y = \dfrac{|x - 1|}{\sqrt{(x - 1)^2 + 9}}$. Thus,

$$y = \arcsin \frac{|x - 1|}{\sqrt{(x - 1)^2 + 9}} = \arcsin \frac{|x - 1|}{\sqrt{x^2 - 2x + 10}}.$$

66. If $\arccos \dfrac{x - 2}{2} = u$, $2 < x < 4$ then $\cos u = \dfrac{x - 2}{2}$.



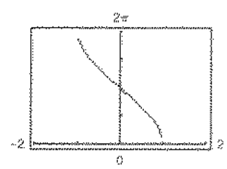
$$\arccos \frac{x - 2}{2} = \arctan \frac{\sqrt{4x - x^2}}{x - 2}, \text{ since } 2 < x < 4$$

67. $y = 2 \arccos x$

Domain: $-1 \le x \le 1$

Range: $0 \le y \le 2\pi$

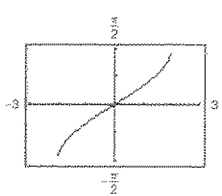
Vertical stretch of $f(x) = \arccos x$



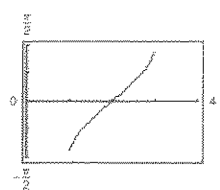
68. $y = \arcsin \dfrac{x}{2}$

Domain: $-2 \le x \le 2$

Range: $-\dfrac{\pi}{2} \le y \le \dfrac{\pi}{2}$



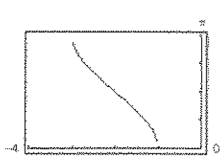
69. The graph of $f(x) = \arcsin(x - 2)$ is a horizontal translation of the graph of $y = \arcsin x$ by two units.



70. $g(t) = \arccos(t + 2)$

Domain: $-3 \le t \le -1$

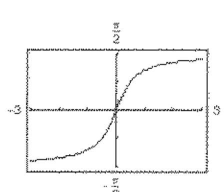
This is the graph of $y = \arccos t$ shifted two units to the left.



71. $f(x) = \arctan 2x$

Domain: all real numbers

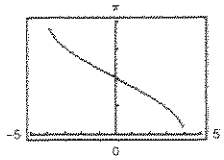
Range: $-\dfrac{\pi}{2} < y < \dfrac{\pi}{2}$



72. $f(x) = \arccos \dfrac{x}{4}$

Domain: $[-4, 4]$

Range: $[0, \pi]$

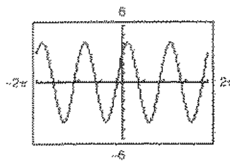


73. $f(t) = 3 \cos 2t + 3 \sin 2t$

$$= \sqrt{3^2 + 3^2} \sin\left(2t + \arctan \frac{3}{3}\right)$$

$$= 3\sqrt{2} \sin(2t + \arctan 1)$$

$$= 3\sqrt{2} \sin\left(2t + \frac{\pi}{4}\right)$$



The graphs are the same.

74. $f(t) = 4 \cos \pi t + 3 \sin \pi t$

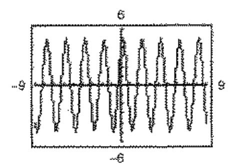
$$= \sqrt{4^2 + 3^2} \sin\left(\pi t + \arctan \frac{4}{3}\right)$$

$$= 5 \sin\left(\pi t + \arctan \frac{4}{3}\right)$$

The graph suggests that

$$A \cos \omega t + B \sin \omega t = \sqrt{A^2 + B^2} \sin\left(\omega t + \arctan \frac{A}{B}\right)$$

is true.



75. As $x \to 1^-$, $\arcsin x \to \dfrac{\pi}{2}$.

76. As $x \to 1^-$, $\arccos x \to 0$.

77. As $x \to \infty$, $\arctan x \to \dfrac{\pi}{2}$.

78. As $x \to -1^+$, $\arcsin x \to -\dfrac{\pi}{2}$.

79. As $x \to -1^+$, $\arccos x \to \pi$.

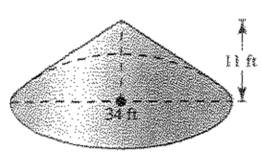
80. As $x \to -\infty$, $\arctan x \to -\dfrac{\pi}{2}$.

81. (a) $\sin \theta = \dfrac{10}{s} \implies \theta = \arcsin\left(\dfrac{10}{s}\right)$

(b) $s = 52$: $\theta = \arcsin\left(\dfrac{10}{52}\right) \approx 0.1935$, $(\approx 11.1°)$

$s = 26$: $\theta = \arcsin\left(\dfrac{10}{26}\right) \approx 0.3948$, $(\approx 22.6°)$

82. (a)



(b) $\tan \theta = \dfrac{11}{17} \implies \theta \approx 0.5743$ or $32.9°$

(c) $\tan(0.5743) = \dfrac{h}{20} \implies h = 20 \tan(0.5743)$

≈ 12.94 feet

83. (a) $\tan \theta = \dfrac{s}{750}$

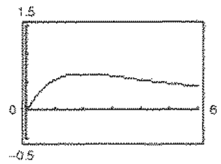
$$\theta = \arctan\left(\frac{s}{750}\right)$$

(b) When $s = 400$,

$$\theta = \arctan\left(\frac{400}{750}\right) \approx 0.49 \text{ radian}, \ (\approx 28°).$$

When $s = 1600$, $\theta \approx 1.13$ radians, $(\approx 65°)$.

84. $\beta = \arctan \dfrac{3x}{x^2+4}, x > 0$

(a) 

(b) β is maximum when $x = 2$ feet.

(c) The graph has a horizontal asymptote at $\beta = 0$. As the camera moves further from the picture, the angle subtended by the camera approaches 0.

85. (a) $\tan\theta = \dfrac{6}{x}$

$$\theta = \arctan\left(\frac{6}{x}\right)$$

(b) When $x = 10$,

$$\theta = \arctan\left(\frac{6}{10}\right) \approx 0.54 \text{ radian}, \ (\approx 31^\circ).$$

When $x = 3$,

$$\theta = \arctan\left(\frac{6}{3}\right) \approx 1.11 \text{ radians}, \ (\approx 63^\circ).$$

86. (a) $\tan\theta = \dfrac{x}{20}$

$$\theta = \arctan\frac{x}{20}$$

(b) When $x = 5$,

$$\theta = \arctan\frac{5}{20} \approx 14.0^\circ, \ (0.24 \text{ rad}).$$

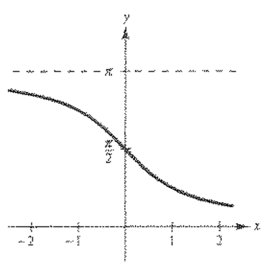
When $x = 12$,

$$\theta = \arctan\frac{12}{20} \approx 31.0^\circ, \ (0.54 \text{ rad}).$$

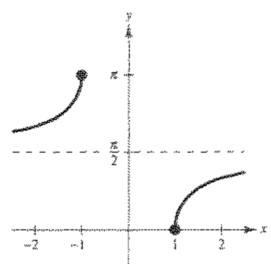
87. False. $\arcsin\dfrac{1}{2} = \dfrac{\pi}{6}$

88. False. $\tan x = \dfrac{\sin x}{\cos x}$

89. $y = \operatorname{arccot} x$ if and only if $\cot y = x$, $-\infty < x < \infty$ and $0 < y < \pi$.



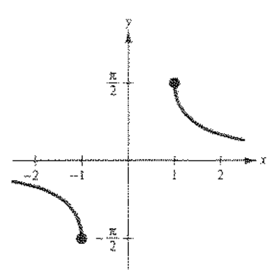
90. $y = \operatorname{arcsec} x$ if and only if $\sec y = x$ where $x \le -1 \cup x \ge 1$ and $0 \le y < \pi/2$ and $\pi/2 < y \le \pi$. The domain of $y = \operatorname{arcsec} x$ is $(-\infty, -1] \cup [1, \infty)$ and the range is $[0, \pi/2) \cup (\pi/2, \pi]$.



91. $y = \operatorname{arccsc} x$ if and only if $\csc y = x$.

Domain: $(-\infty, -1] \cup [1, \infty)$

Range: $\left[-\dfrac{\pi}{2}, 0\right) \cup \left(0, \dfrac{\pi}{2}\right]$



92. (a) $y = \operatorname{arccot} x$ if and only if $x = \cot y$, $-\infty < x < \infty$ and $0 < y < \pi$.

Thus, $\dfrac{1}{x} = \tan y$ and $y = \arctan\left(\dfrac{1}{x}\right)$.

Hence, graph $y = \begin{cases} \pi + \arctan(1/x), & x < 0 \\ \pi/2, & x = 0. \\ \arctan(1/x), & x > 0 \end{cases}$

(b) $y = \operatorname{arcsec} x$ if and only if $x = \sec y$, $x \le -1$ or $x \ge 1$, and $0 \le y < \dfrac{\pi}{2}$ or $\dfrac{\pi}{2} < y < \pi$.

Thus, $\dfrac{1}{x} = \cos y$ and $y = \arccos\left(\dfrac{1}{x}\right)$. Hence graph $y = \arccos\dfrac{1}{x}$, $x \in (-\infty, -1] \cup [1, \infty)$.

(c) $y = \operatorname{arccsc} x$ if and only if $x = \csc y$, $x \le -1$ or $x \ge 1$, and $-\dfrac{\pi}{2} \le y < 0$ or $0 < y \le \dfrac{\pi}{2}$.

Thus, $\dfrac{1}{x} = \sin y$ and $y = \arcsin\left(\dfrac{1}{x}\right)$. Hence, graph $y = \arcsin\left(\dfrac{1}{x}\right)$, $x \in (-\infty, -1] \cup [1, \infty)$.

93. $y = \operatorname{arcsec}\sqrt{2} \implies \sec y = \sqrt{2}$ and $0 \le y < \dfrac{\pi}{2} \cup \dfrac{\pi}{2} < y \le \pi \implies y = \dfrac{\pi}{4}$

94. $y = \operatorname{arcsec} 1 \implies \sec y = 1$ and $0 \le y < \dfrac{\pi}{2} \cup \dfrac{\pi}{2} < y \le \pi \implies y = 0$

95. $y = \operatorname{arccot}\left(-\sqrt{3}\right) \implies \cot y = -\sqrt{3}$ and $0 < y < \pi \implies y = \dfrac{5\pi}{6}$

96. $y = \operatorname{arccsc} 2 \implies \csc y = 2$ and $-\dfrac{\pi}{2} \le y < 0 \cup 0 < y \le \dfrac{\pi}{2} \implies y = \dfrac{\pi}{6}$

97. Let $y = \arcsin(-x)$. Then,

$$\begin{aligned} \sin y &= -x \\ -\sin y &= x \\ \sin(-y) &= x \\ -y &= \arcsin x \\ y &= -\arcsin x. \end{aligned}$$

Therefore, $\arcsin(-x) = -\arcsin x$.

98.

$$\begin{aligned} y &= \arctan(-x) \\ \tan y &= -x,\ -\frac{\pi}{2} < y < \frac{\pi}{2} \\ -\tan y &= x \\ \tan(-y) &= x,\ -\frac{\pi}{2} < -y < \frac{\pi}{2} \\ \arctan(\tan(-y)) &= \arctan x \\ -y &= \arctan x \\ y &= -\arctan x \end{aligned}$$

99. $\arcsin x + \arccos x = \dfrac{\pi}{2}$

Let $\alpha = \arcsin x \implies \sin\alpha = x$.

Let $\beta = \arccos x \implies \cos\beta = x$.

Hence, $\sin a = \cos\beta \implies a$ and β are complementary angles $\implies a + \beta = \dfrac{\pi}{2} \implies \arcsin x + \arccos x = \dfrac{\pi}{2}$.

100. Area $=$ arctan $b -$ arctan a

(a) $a = 0, b = 1$

Area $= \arctan 1 - \arctan 0$

$$= \frac{\pi}{4} - 0 = \frac{\pi}{4} \approx 0.785$$

(b) $a = -1, b = 1$

Area $= \arctan 1 - \arctan(-1)$

$$= \frac{\pi}{4} - \left(-\frac{\pi}{4}\right) = \frac{\pi}{2} \approx 1.571$$

(c) $a = 0, b = 3$

Area $= \arctan 3 - \arctan 0$

$$\approx 1.25 - 0 = 1.25$$

$$= 1.25$$

(d) $a = -1, b = 3$

Area $= \arctan 3 - \arctan(-1)$

$$\approx 1.25 - \left(-\frac{\pi}{4}\right) \approx 2.03$$

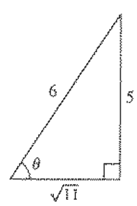
101. $\dfrac{4}{4\sqrt{2}} = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$

102. $\dfrac{2}{\sqrt{3}} = \dfrac{2\sqrt{3}}{\sqrt{3}\sqrt{3}} = \dfrac{2\sqrt{3}}{3}$

103. $\dfrac{2\sqrt{3}}{6} = \dfrac{\sqrt{3}}{3}$

104. $\dfrac{5\sqrt{5}}{2\sqrt{10}} = \dfrac{5\sqrt{5}}{2\sqrt{2}\sqrt{5}} = \dfrac{5}{2\sqrt{2}} = \dfrac{5\sqrt{2}}{4}$

105. $\sin\theta = \dfrac{5}{6}$



Adjacent side: $\sqrt{6^2 - 5^2} = \sqrt{11}$

$\cos\theta = \dfrac{\sqrt{11}}{6}$

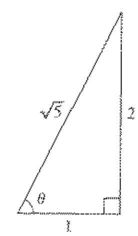
$\tan\theta = \dfrac{5}{\sqrt{11}} = \dfrac{5\sqrt{11}}{11}$

$\csc\theta = \dfrac{6}{5}$

$\sec\theta = \dfrac{6}{\sqrt{11}} = \dfrac{6\sqrt{11}}{11}$

$\cot\theta = \dfrac{\sqrt{11}}{5}$

106. $\tan\theta = 2, 0 < \theta < \dfrac{\pi}{2}$



$\sin\theta = \dfrac{2}{\sqrt{5}} = \dfrac{2\sqrt{5}}{5}$

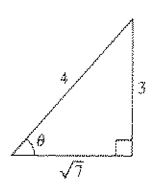
$\cos\theta = \dfrac{\sqrt{5}}{5}$

$\csc\theta = \dfrac{\sqrt{5}}{2}$

$\sec\theta = \sqrt{5}$

$\cot\theta = \dfrac{1}{2}$

107. $\sin\theta = \dfrac{3}{4}$



Adjacent side: $\sqrt{16 - 9} = \sqrt{7}$

$\cos\theta = \dfrac{\sqrt{7}}{4}$

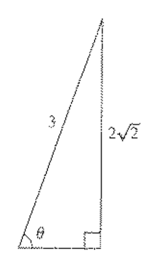
$\tan\theta = \dfrac{3}{\sqrt{7}} = \dfrac{3\sqrt{7}}{7}$

$\csc\theta = \dfrac{4}{3}$

$\sec\theta = \dfrac{4}{\sqrt{7}} = \dfrac{4\sqrt{7}}{7}$

$\cot\theta = \dfrac{\sqrt{7}}{3}$

108. $\sec\theta = 3, 0 < \theta < \dfrac{\pi}{2}$



$\sin\theta = \dfrac{2\sqrt{2}}{3}$

$\cos\theta = \dfrac{1}{3}$

$\tan\theta = 2\sqrt{2}$

$\csc\theta = \dfrac{3}{2\sqrt{2}} = \dfrac{3\sqrt{2}}{4}$

$\cot\theta = \dfrac{1}{2\sqrt{2}} = \dfrac{\sqrt{2}}{4}$

Section 4.8 Applications and Models

- You should be able to solve right triangles.
- You should be able to solve right triangle applications.
- You should be able to solve applications of simple harmonic motion: $d = a \sin wt$ or $d = a \cos wt$.

Vocabulary Check

1. elevation, depression
2. bearing
3. harmonic motion

1. Given: $A = 30°, b = 10$

$B = 90° - 30° = 60°$

$\tan A = \frac{a}{b} \implies a = b \tan A = 10 \tan 30° \approx 5.77$

$\cos A = \frac{b}{c} \implies c = \frac{b}{\cos A} = \frac{10}{\cos 30°} \approx 11.55$

2. Given: $B = 60°, c = 15$

$A = 90° - 60° = 30°$

$\sin B = \frac{b}{c} \implies b = c \sin B = 15 \sin 60° = \frac{15\sqrt{3}}{2} \approx 12.99$

$\cos B = \frac{a}{c} \implies a = c \cos B = 15 \cos 60° = \frac{15}{2} = 7.50$

3. Given: $B = 71°, b = 14$

$\tan B = \frac{b}{a} \implies a = \frac{b}{\tan B} = \frac{14}{\tan 71°} \approx 4.82$

$\sin B = \frac{b}{c} \implies c = \frac{b}{\sin B} = \frac{14}{\sin 71°} \approx 14.81$

$A = 90° - 71° = 19°$

4. Given: $A = 7.4°, a = 20.5$

$B = 90° - 7.4° = 82.6°$

$\tan A = \frac{a}{b} \implies b = \frac{a}{\tan A} = \frac{20.5}{\tan 7.4°} \approx 157.84$

$\sin A = \frac{a}{c} \implies c = \frac{a}{\sin A} = \frac{20.5}{\sin 7.4°} \approx 159.17$

5. Given: $a = 6, b = 12$

$c^2 = a^2 + b^2 \implies c = \sqrt{36 + 144} \approx 13.42$

$\tan A = \frac{a}{b} = \frac{6}{12} = \frac{1}{2} \implies A = \arctan \frac{1}{2} \approx 26.57°$

$B = 90° - 26.57° = 63.43°$

6. Given: $a = 25, c = 45$

$b = \sqrt{c^2 - a^2} = \sqrt{1400} = 10\sqrt{14} \approx 37.42$

$\sin A = \frac{a}{c} \implies A = \arcsin\left(\frac{25}{45}\right) \approx 33.75°$

$\cos B = \frac{a}{c} \implies B = \arccos\left(\frac{25}{45}\right) \approx 56.25°$

7. Given: $b = 16, c = 54$

$$a = \sqrt{c^2 - b^2} = \sqrt{2660} \approx 51.58$$

$$\cos A = \frac{b}{c} = \frac{16}{54} \implies A = \arccos\left(\frac{16}{54}\right) \approx 72.76^\circ$$

$$B = 90^\circ - 72.76^\circ = 17.24^\circ$$

8. Given: $b = 1.32, c = 18.9$

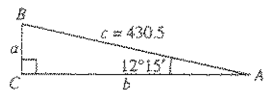
$$a = \sqrt{c^2 - b^2} = \sqrt{355.4676} \approx 18.85$$

$$\cos A = \frac{b}{c} = \frac{1.32}{18.9} \implies A = \arccos\left(\frac{1.32}{18.9}\right) \approx 86.00^\circ$$

$$\sin B = \frac{b}{c} \implies B = \arcsin\left(\frac{1.32}{18.9}\right) \approx 4.00^\circ$$

9. $A = 12^\circ\ 15',\ c = 430.5$

$$B = 90^\circ - 12^\circ\ 15' = 77^\circ\ 45'$$



$$\sin 12^\circ\ 15' = \frac{a}{430.5}$$

$$a = 430.5 \sin 12^\circ\ 15' \approx 91.34$$

$$\cos 12^\circ\ 15' = \frac{b}{430.5}$$

$$b = 430.5 \cos 12^\circ\ 15' \approx 420.70$$

10. Given: $B = 65^\circ\ 12', a = 145.5$

$$A = 90^\circ - 65^\circ\ 12' = 24^\circ\ 48'$$

$$\cos B = \frac{a}{c} \implies c = \frac{a}{\cos B} = \frac{145.5}{\cos(65^\circ\ 12')} \approx 346.88$$

$$\tan B = \frac{b}{a} \implies b = a \tan B = 145.5 \tan(65^\circ\ 12') \approx 314.89$$

11. $\tan \theta = \dfrac{h}{b/2}$

$$h = \frac{1}{2}b \tan \theta$$

$$h = \frac{1}{2}(8) \tan 52^\circ \approx 5.12 \text{ in.}$$

12. $\tan \theta = \dfrac{h}{b/2}$

$$h = \frac{1}{2}b \tan \theta$$

$$h = \frac{1}{2}(12) \tan 18^\circ \approx 1.95 \text{ meters}$$

13. $\tan \theta = \dfrac{h}{b/2}$

$$h = \frac{1}{2}b \tan \theta$$

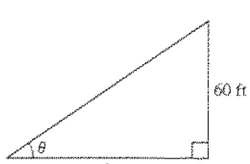
$$h = \frac{1}{2}(18.5) \tan 41.6^\circ \approx 8.21 \text{ ft}$$

14. $\tan \theta = \dfrac{h}{b/2}$

$$h = \frac{1}{2}b \tan \theta$$

$$h = \frac{1}{2}(3.26) \tan 72.94^\circ \approx 5.31 \text{ cm}$$

15. (a)



(b) $\tan \theta = \dfrac{60}{L}$

$$L = \frac{60}{\tan \theta}$$

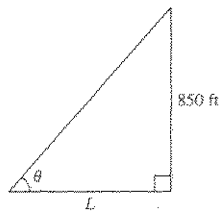
$$= 60 \cot \theta$$

(c)

θ	10°	20°	30°	40°	50°
L	340	165	104	72	50

(d) No, the shadow lengths do not increase in equal increments. The cotangent function is not linear.

16. (a)



(b) $\tan\theta = \dfrac{850}{L}$

$$L = \frac{850}{\tan\theta}$$

$$= 850\cot\theta$$

(c)

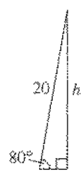
θ	10°	20°	30°	40°	50°
L	4821	2335	1472	1013	713

(d) No, the cotangent function is not a linear function.

17. $\sin 80^\circ = \dfrac{h}{20}$

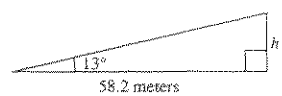
$$h = 20\sin 80^\circ$$

$$\approx 19.70 \text{ feet}$$



18. $\tan(13^\circ) = \dfrac{h}{58.2}$

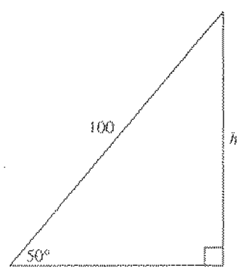
$$h = 58.2\tan(13^\circ) \approx 13.44 \text{ meters}$$



19. $\sin 50^\circ = \dfrac{h}{100}$

$$h = 100\sin 50^\circ$$

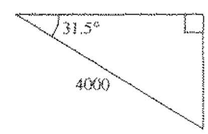
$$\approx 76.6 \text{ feet}$$



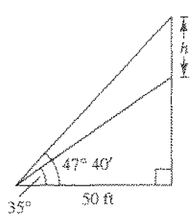
20. $\sin 31.5^\circ = \dfrac{x}{4000}$

$$x = 4000\sin 31.5^\circ$$

$$\approx 2089.99 \text{ feet}$$



21. (a)



(b) Let the height of the church $= x$ and the height of the church and steeple $= y$. Then:

$$\tan 35^\circ = \frac{x}{50} \text{ and } \tan 47^\circ\,40' = \frac{y}{50}$$

$$x = 50\tan 35^\circ \text{ and } y = 50\tan 47^\circ\,40'$$

$$h = y - x = 50(\tan 47^\circ\,40' - \tan 35^\circ)$$

(c) $h \approx 19.9$ feet

22. $\tan 28^\circ = \dfrac{a}{100} \implies a = 100\tan 28^\circ$

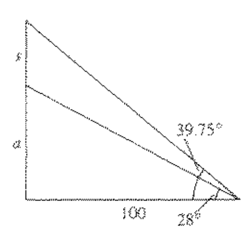
$$\tan 39.75^\circ = \frac{a+s}{100}$$

$$a + s = 100\tan 39.75^\circ$$

$$s = 100\tan 39.75^\circ - a$$

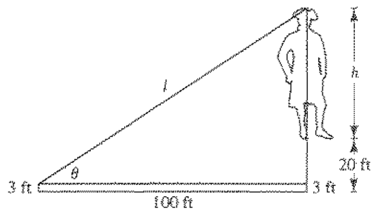
$$= 100\tan 39.75^\circ - 100\tan 28^\circ$$

$$\approx 30 \text{ feet}$$



23. (a) $l^2 = 100^2 + (20 - 3 + h)^2$

$$l = \sqrt{100^2 + (17 + h)^2}$$
$$= \sqrt{h^2 + 34h + 10{,}289}$$



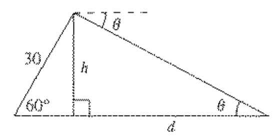
(b) $\cos\theta = \dfrac{100}{l} \implies \theta = \arccos\left(\dfrac{100}{l}\right)$

(c) If $\theta = 35°$, then $l = \dfrac{100}{\cos 35°} \approx 122.077$. Then

$$(17 + h)^2 + 100^2 = l^2$$
$$(17 + h)^2 = l^2 - 100^2 \approx 4902.906$$
$$17 + h \approx 70.02$$
$$h \approx 53.02 \text{ feet}$$
$$\approx 53 \text{ feet}, \frac{1}{4} \text{ inch.}$$

24. (a) $\sin 60° = \dfrac{h}{30} \implies h = 30\sin 60°$

$$= 15\sqrt{3} \approx 25.98 \text{ ft}$$



(b) $\tan\theta = \dfrac{h}{d} = \dfrac{15\sqrt{3}}{d} \approx \dfrac{25.98}{d}$

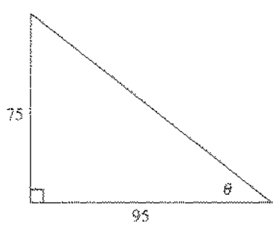
(c) $25° \le \theta \le 30°$

$$\tan 25° \le \tan\theta \le \tan 30°$$
$$\tan 25° \le \frac{h}{d} \le \tan 30°$$
$$\frac{h}{\tan 25°} \ge d \ge \frac{h}{\tan 30°}$$
$$55.72 \ge d \ge 45.00$$

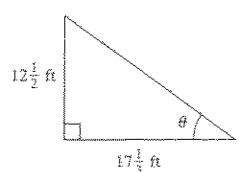
d is in the interval $(45.0, 55.72)$.

25. $\tan\theta = \dfrac{75}{95}$

$$\theta = \arctan\frac{15}{19} \approx 38.29°$$



26. (a)

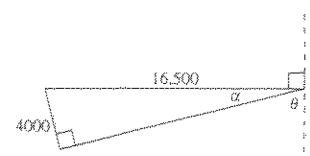


(b) $\tan\theta = \dfrac{12\frac{1}{2}}{17\frac{1}{3}}$

(c) $\theta = \arctan\dfrac{12\frac{1}{2}}{17\frac{1}{3}} \approx 35.8°$

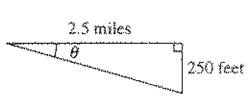
27. $\sin\alpha = \dfrac{4000}{16{,}500} \implies \alpha \approx 14.03°$

$$\theta \approx 90° - \alpha \approx 75.97°$$



28. $\tan\theta = \dfrac{250}{2.5(5280)}$

$$\theta = \arctan\left(\frac{250}{2.5(5280)}\right) \approx 1.09°$$

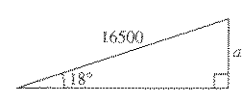


29. Since the airplane speed is

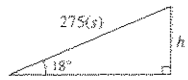
$$\left(275\,\frac{\text{ft}}{\text{sec}}\right)\left(60\,\frac{\text{sec}}{\text{min}}\right) = 16{,}500\,\frac{\text{ft}}{\text{min}},$$

after one minute its distance travelled is 16,500 feet.

$$\sin 18° = \frac{a}{16{,}500}$$
$$a = 16{,}500 \sin 18°$$
$$\approx 5099 \text{ ft}$$



30. $\sin 18° = \dfrac{h}{275(s)}$

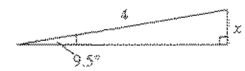


$s = \dfrac{h}{275 \sin 18°}$

If $h = 10{,}000$, $s = \dfrac{10{,}000}{275 \sin 18°} \approx 117.7$ seconds.

If $h = 16{,}000$, $s = \dfrac{16{,}000}{275 \sin 18°} \approx 188.3$ seconds.

31. $\sin 9.5° = \dfrac{x}{4}$

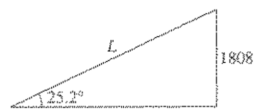


$x = 4 \sin 9.5°$

≈ 0.66 mile

32. $\sin(25.2°) = \dfrac{1808}{L}$

$\Longrightarrow L = \dfrac{1808}{\sin(25.2°)} \approx 4246.3$ feet

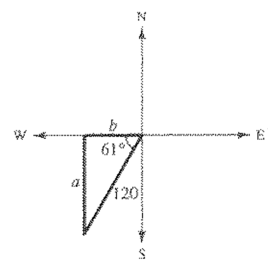


33. $90° - 29° = 61°$

$(20)(6) = 120$ nautical miles

$\sin 61° = \dfrac{a}{120} \Longrightarrow a = 120 \sin 61° \approx 104.95$ nautical miles

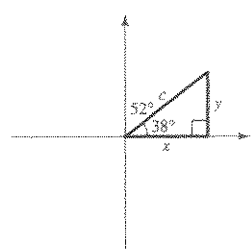
$\cos 61° = \dfrac{b}{120} \Longrightarrow b = 120 \cos 61° \approx 58.18$ nautical miles



34. $c = 600(1.5) = 900$

$y = 900 \sin 38° \approx 554.1$ miles

$x = 900 \cos 38° \approx 709.2$ miles



35. $\theta = 32°$, $\phi = 68°$ *Note: ABC* forms a right triangle.

(a) $\alpha = 90° - 32° = 58°$

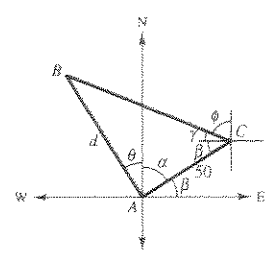
Bearing from A to C: N 58° E

(b) $\beta = \theta = 32°$

$\gamma = 90° - \phi = 22°$

$C = \beta + \gamma = 54°$

$\tan C = \dfrac{d}{50} \Longrightarrow \tan 54° = \dfrac{d}{50} \Longrightarrow d \approx 68.82$ m



36. $\tan 14° = \dfrac{d}{x} \implies x = d \cot 14°$

$$\tan 34° = \frac{d}{y} \implies \frac{d}{30 - x} = \frac{d}{30 - d\cot 14°}$$

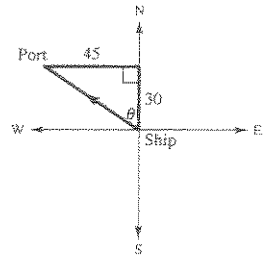
$$\cot 34° = \frac{30 - d\cot 14°}{d}$$

$$d\cot 34° = 30 - d\cot 14°$$

$$d = \frac{30}{\cot 34° + \cot 14°} \approx 5.46 \text{ kilometers}$$

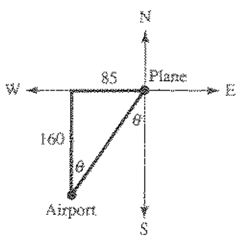
37. $\tan\theta = \dfrac{45}{30} = \dfrac{3}{2} \implies \theta \approx 56.31°$

Bearing: N 56.31° W



38. $\tan\theta = \dfrac{85}{160} \implies \theta \approx 27.98°$

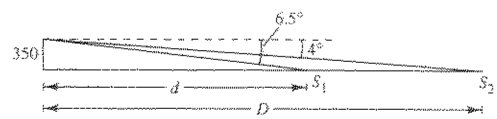
Bearing: S 27.98° W



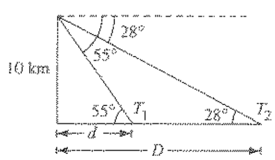
39. $\tan 6.5° = \dfrac{350}{d} \implies d \approx 3071.91$ ft

$$\tan 4° = \frac{350}{D} \implies D \approx 5005.23 \text{ ft}$$

Distance between ships: $D - d \approx 1933.3$ ft



40.



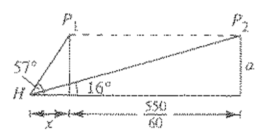
$$\cot 55° = \frac{d}{10} \implies d \approx 7 \text{ kilometers}$$

$$\cot 28° = \frac{D}{10} \implies D \approx 18.8 \text{ kilometers}$$

Distance between towns:

$$D - d = 18.8 - 7 = 11.8 \text{ kilometers}$$

41. $\tan 57° = \dfrac{a}{x} \implies x = a\cot 57°$



$$\tan 16° = \frac{a}{x + (55/6)}$$

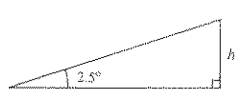
$$\tan 16° = \frac{a}{a\cot 57° + (55/6)}$$

$$\cot 16° = \frac{a\cot 57° + (55/6)}{a}$$

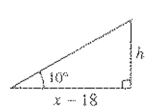
$$a\cot 16° - a\cot 57° = \frac{55}{6}$$

$$\implies a \approx 3.23 \text{ miles} \approx 17{,}054 \text{ feet}$$

42. $\tan 2.5° = \dfrac{h}{x}, \tan 10° = \dfrac{h}{x - 18}$



not drawn to scale



not drawn to scale

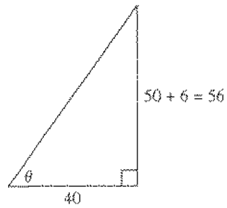
$$x = \frac{h}{\tan 2.5°}, x = \frac{h}{\tan 10°} + 18$$

$$\frac{h}{\tan 2.5°} = \frac{h}{\tan 10°} + 18 = \frac{h + 18\tan 10°}{\tan 10°}$$

$$h\tan 10° = h\tan 2.5° + 18(\tan 10°)(\tan 2.5°)$$

$$h = \frac{18(\tan 10°)(\tan 2.5°)}{\tan 10° - \tan 2.5°} \approx 1.04 \text{ miles} \approx 5515 \text{ feet}$$

43. (a)



$$\theta = \arctan\frac{56}{40} \approx 54.5^\circ$$

Similarly for the back row,

$$\theta = \arctan\frac{56}{150} \approx 20.5^\circ.$$

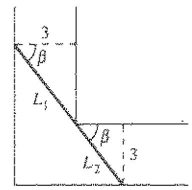
(b) For 45°, you need to be 56 feet away since

$$\arctan\frac{56}{56} = 45^\circ.$$

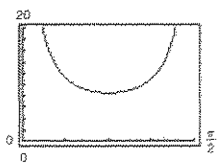
44. (a) Using the two triangles with acute angle β,

$$\cos\beta = \frac{3}{L_1} \text{ and } \sin\beta = \frac{3}{L_2}.$$

Hence, $L = L_1 + L_2 = 3\sec\beta + 3\csc\beta$.



(b)



(c) The least value is $\beta \approx 0.785$. $\left(\beta = \dfrac{\pi}{4}\right)$

45. L_1: $3x - 2y = 5 \implies y = \frac{3}{2}x - \frac{5}{2} \implies m_1 = \frac{3}{2}$

L_2: $x + y = 1 \implies y = -x + 1 \implies m_2 = -1$

$$\tan\alpha = \left|\frac{-1 - (3/2)}{1 + (-1)(3/2)}\right| = \left|\frac{-5/2}{-1/2}\right| = 5$$

$$\alpha = \arctan 5 \approx 78.7^\circ$$

46. $L_1 = 2x + y = 8 \implies m_1 = -2$

$L_2 = x - 5y = -4 \implies m_2 = \frac{1}{5}$

$$\tan\alpha = \left|\frac{m_2 - m_1}{1 + m_2m_1}\right|$$

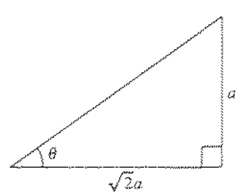
$$\alpha = \arctan\left|\frac{m_2 - m_1}{1 + m_2m_1}\right|$$

$$= \arctan\left|\frac{\frac{1}{5} - (-2)}{1 + \frac{1}{5}(-2)}\right| = \arctan\left(3\tfrac{2}{3}\right) \approx 74.7^\circ$$

47. The diagonal of the base has a length of $\sqrt{a^2 + a^2} = \sqrt{2}a$. Now, we have:

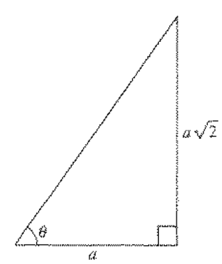
$$\tan\theta = \frac{a}{\sqrt{2}a} = \frac{1}{\sqrt{2}}$$

$$\theta = \arctan\frac{1}{\sqrt{2}} \approx 35.3^\circ$$



48. $\tan\theta = \dfrac{a\sqrt{2}}{a} = \sqrt{2}$

$$\theta = \arctan\sqrt{2} \approx 54.7^\circ$$



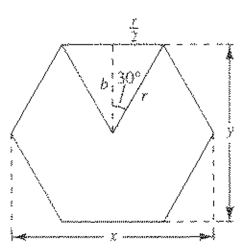
49. $\cos 30^\circ = \dfrac{b}{r}$

$$b = \cos 30^\circ r$$

$$b = \frac{\sqrt{3}r}{2}$$

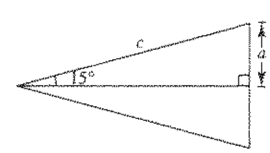
$$y = 2b$$

$$= 2\left(\frac{\sqrt{3}r}{2}\right) = \sqrt{3}r$$



50. $c = \dfrac{35}{2} = 17.5$

$$\sin 15^\circ = \frac{a}{c}$$



$$a = c\sin 15^\circ = 17.5\sin 15^\circ \approx 4.53$$

Distance $= 2a \approx 9.06$ centimeters

51. $d = 0$ when $t = 0$, $a = 8$, period $= 2$

Use $d = a \sin \omega t$ since $d = 0$ when $t = 0$.

$\dfrac{2\pi}{\omega} = 2 \implies \omega = \pi$

Thus, $d = 8 \sin \pi t$.

52. Displacement at $t = 0$ is $0 \implies d = a \sin \omega t$.

Amplitude: $|a| = 3$

Period: $\dfrac{2\pi}{\omega} = 6 \implies \omega = \dfrac{\pi}{3}$

$d = 3 \sin\left(\dfrac{\pi t}{3}\right)$

53. $d = 3$ when $t = 0$, $a = 3$, period $= 1.5$

Use $d = a \cos \omega t$ since $d = 3$ when $t = 0$.

$\dfrac{2\pi}{\omega} = 1.5 \implies \omega = \dfrac{4}{3}\pi$

Thus, $d = 3 \cos\left(\dfrac{4}{3}\pi t\right)$.

54. Displacement at $t = 0$ is $2 \implies d = a \cos \omega t$

Amplitude: $|a| = 2$

Period: $\dfrac{2\pi}{\omega} = 10 \implies \omega = \dfrac{\pi}{5}$

$d = 2 \cos\left(\dfrac{\pi t}{5}\right)$

55. $d = 4 \cos 8\pi t$

(a) Maximum displacement $=$ amplitude $= 4$

(b) Frequency $= \dfrac{\omega}{2\pi} = \dfrac{8\pi}{2\pi}$

$= 4$ cycles per unit of time

(c) $d = 4 \cos(8\pi(5)) = 4$

(d) $8\pi t = \dfrac{\pi}{2} \implies t = \dfrac{1}{16}$

56. $d = \dfrac{1}{2} \cos 20\pi t$

(a) Maximum displacement: $|a| = \left|\dfrac{1}{2}\right| = \dfrac{1}{2}$

(b) Frequency: $\dfrac{\omega}{2\pi} = \dfrac{20\pi}{2\pi} = 10$

(c) When $t = 5$, $d = \dfrac{1}{2} \cos[20\pi(5)] = \dfrac{1}{2}$.

(d) Least positive value for t for which $d = 0$:

$$\frac{1}{2} \cos 20\pi t = 0$$

$$\cos 20\pi t = 0$$

$$20\pi t = \frac{\pi}{2}$$

$$t = \frac{\pi}{2} \cdot \frac{1}{20\pi} = \frac{1}{40}$$

57. $d = \dfrac{1}{16} \sin 140\pi t$

(a) Maximum displacement $=$ amplitude $= \dfrac{1}{16}$

(b) Frequency $= \dfrac{\omega}{2\pi} = \dfrac{140\pi}{2\pi}$

$= 70$ cycles per unit of time

(c) $d = 0$

(d) $140\pi t = \pi \implies t = \dfrac{1}{140}$

58. $d = \frac{1}{64}\sin 792\pi t$

(a) Maximum displacement: $|a| = \left|\frac{1}{64}\right| = \frac{1}{64}$

(b) Frequency: $\frac{\omega}{2\pi} = \frac{792\pi}{2\pi} = 396$

(c) When $t = 5$, $d = \frac{1}{64}\sin[792\pi(5)] = 0$.

(d) Least positive value for t for which $d = 0$:

$$\frac{1}{64}\sin 792\pi t = 0$$
$$\sin 792\pi t = 0$$
$$792\pi t = \pi$$
$$t = \frac{\pi}{792\pi} = \frac{1}{792}$$

59. $d = a\sin\omega t$

$$\text{Period} = \frac{2\pi}{\omega} = \frac{1}{\text{frequency}}$$
$$\frac{2\pi}{\omega} = \frac{1}{264}$$
$$\omega = 2\pi(264) = 528\pi$$

60. At $t = 0$, buoy is at its high point $\Longrightarrow$ $d = a\cos\omega t$.

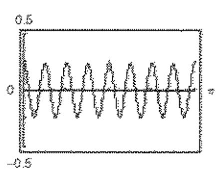
Distance from high to low $= 2|a| = 3.5$

$$|a| = \frac{7}{4}$$

Returns to high point every 10 seconds:

$$\text{Period} = \frac{2\pi}{\omega} = 10 \Longrightarrow \omega = \frac{\pi}{5}$$
$$d = \frac{7}{4}\cos\frac{\pi t}{5}$$

61. $y = \frac{1}{4}\cos 16t,\ t > 0$

(a) 

(b) Period: $\frac{2\pi}{16} = \frac{\pi}{8}$ seconds

(c) $\frac{1}{4}\cos 16t = 0$ when

$$16t = \frac{\pi}{2} \Longrightarrow t = \frac{\pi}{32} \text{ seconds.}$$

62. (a)

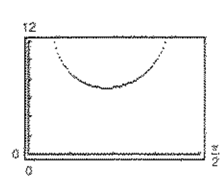
θ	L_1	L_2	$L_1 + L_2$
0.1	$\frac{2}{\sin 0.1}$	$\frac{3}{\cos 0.1}$	23.0
0.2	$\frac{2}{\sin 0.2}$	$\frac{3}{\cos 0.2}$	13.1
0.3	$\frac{2}{\sin 0.3}$	$\frac{3}{\cos 0.3}$	9.9
0.4	$\frac{2}{\sin 0.4}$	$\frac{3}{\cos 0.4}$	8.4

(b)

0.5	$\frac{2}{\sin 0.5}$	$\frac{3}{\cos 0.5}$	7.6
0.6	$\frac{2}{\sin 0.6}$	$\frac{3}{\cos 0.6}$	7.2
0.7	$\frac{2}{\sin 0.7}$	$\frac{3}{\cos 0.7}$	7.0
0.8	$\frac{2}{\sin 0.8}$	$\frac{3}{\cos 0.8}$	7.1

The minimum length of the elevator is 7.0 meters.

(c) $L = L_1 + L_2 = \frac{2}{\sin\theta} + \frac{3}{\cos\theta}$

(d) 

From the graph, it appears that the minimum length is 7.0 meters, which agrees with the estimate of part (b).

63. (a), (b)

Base 1	Base 2	Altitude	Area
8	$8 + 16 \cos 10°$	$8 \sin 10°$	22.1
8	$8 + 16 \cos 20°$	$8 \sin 20°$	42.5
8	$8 + 16 \cos 30°$	$8 \sin 30°$	59.7
8	$8 + 16 \cos 40°$	$8 \sin 40°$	72.7
8	$8 + 16 \cos 50°$	$8 \sin 50°$	80.5
8	$8 + 16 \cos 60°$	$8 \sin 60°$	83.1
8	$8 + 16 \cos 70°$	$8 \sin 70°$	80.7

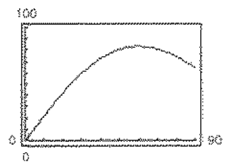
Maximum $\approx$ 83.1 square feet

(c) $A = \frac{1}{2}(b_1 + b_2)h$

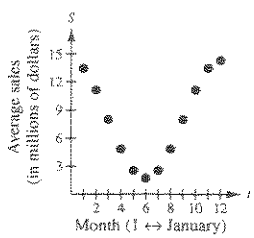
$= \frac{1}{2}[8 + 8 + 16 \cos \theta]8 \sin \theta$

$= 64(1 + \cos \theta) \sin \theta$

(d) Maximum area is approximately 83.1 square feet for $\theta = 60°$.



64. (a)



(b) $a = \frac{1}{2}(14.30 - 1.70) = 6.3$

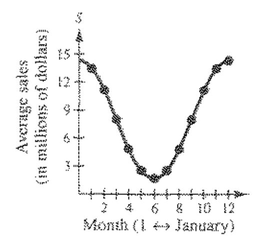
$\frac{2\pi}{b} = 12 \implies b = \frac{\pi}{6}$

Shift: $d = 14.3 - 6.3 = 8$

$S = d + a \cos bt$

$S = 8 + 6.3 \cos\left(\frac{\pi t}{6}\right)$

The model is a good fit.



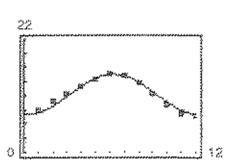
(c) Period: $\frac{2\pi}{\pi/6} = 12$

This corresponds to the 12 months in a year. Since the sales of outerwear is seasonal, this is reasonable.

(d) The amplitude represents the maximum displacement from the average sale of 8 million dollars. Sales are greatest in December (cold weather + holidays) and least in June.

65. $S(t) = 18.09 + 1.41 \sin\left(\frac{\pi t}{6} + 4.60\right)$

(a)



(b) The period is $\frac{2\pi}{(\pi/6)} = 12$ months, which is 1 year.

(c) The amplitude is 1.41. This gives the maximum change in time from the average time (18.09) of sunset.

66. False

It means 24 degrees east of north.

67. False. The other acute angle is $90° - 48.1° = 41.9°$. Then

$\tan(41.9°) = \frac{\text{opp}}{\text{adj}}$

$= \frac{a}{22.56} \implies a = 22.56 \cdot \tan(41.9°).$

68. False

69. $y - 2 = 4(x + 1)$

$4x - y + 6 = 0$

70. $y - 0 = -\frac{1}{2}\left(x - \frac{1}{3}\right)$

$2y = -x + \frac{1}{3}$

$3x + 6y - 1 = 0$

71. Slope $= \dfrac{6 - 2}{-2 - 3} = -\dfrac{4}{5}$

$y - 2 = -\dfrac{4}{5}(x - 3)$

$5y - 10 = -4x + 12$

$4x + 5y - 22 = 0$

72. $m = \dfrac{1/3 + 2/3}{-1/2 - 1/4} = \dfrac{1}{-3/4} = -\dfrac{4}{3}$

$y + \dfrac{2}{3} = -\dfrac{4}{3}\left(x - \dfrac{1}{4}\right)$

$3y + 2 = -4x + 1$

$4x + 3y + 1 = 0$

73. Domain: $(-\infty, \infty)$

74. Domain: $(-\infty, \infty)$

75. Domain: $(-\infty, \infty)$

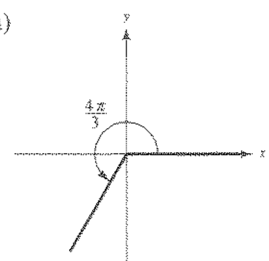
76. Domain: $7 - x \geq 0$ or $x \leq 7$

Review Exercises for Chapter 4

1. 40° or 0.7 radian

2. 250° or 4.4 radians

3. (a)

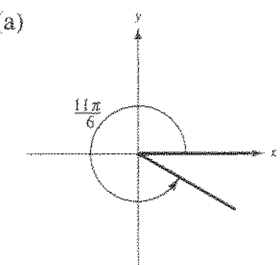


(b) Quadrant III

(c) $\dfrac{4\pi}{3} + 2\pi = \dfrac{10\pi}{3}$

$\dfrac{4\pi}{3} - 2\pi = -\dfrac{2\pi}{3}$

4. (a)

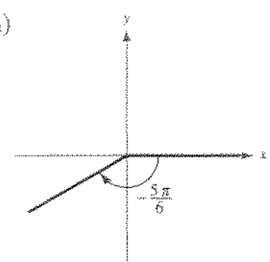


(b) Quadrant IV

(c) $\dfrac{11\pi}{6} + 2\pi = \dfrac{23\pi}{6}$

$\dfrac{11\pi}{6} - 2\pi = -\dfrac{\pi}{6}$

5. (a)

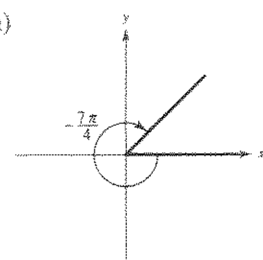


(b) Quadrant III

(c) $-\dfrac{5\pi}{6} + 2\pi = \dfrac{7\pi}{6}$

$-\dfrac{5\pi}{6} - 2\pi = -\dfrac{17\pi}{6}$

6. (a)



(b) Quadrant I

(c) $-\dfrac{7\pi}{4} + 2\pi = \dfrac{\pi}{4}$

$-\dfrac{7\pi}{4} - 2\pi = -\dfrac{15\pi}{4}$

7. Complement: $\dfrac{\pi}{2} - \dfrac{\pi}{8} = \dfrac{3\pi}{8}$

Supplement: $\pi - \dfrac{\pi}{8} = \dfrac{7\pi}{8}$

8. Complement: $\dfrac{\pi}{2} - \dfrac{\pi}{12} = \dfrac{5\pi}{12}$

Supplement: $\pi - \dfrac{\pi}{12} = \dfrac{11\pi}{12}$

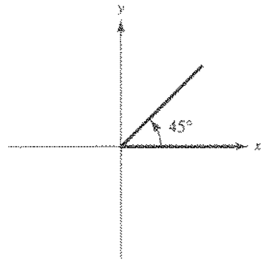
9. Complement: $\dfrac{\pi}{2} - \dfrac{3\pi}{10} = \dfrac{\pi}{5}$

Supplement: $\pi - \dfrac{3\pi}{10} = \dfrac{7\pi}{10}$

10. Complement: $\dfrac{\pi}{2} - \dfrac{2\pi}{21} = \dfrac{17\pi}{42}$

Supplement: $\pi - \dfrac{2\pi}{21} = \dfrac{19\pi}{21}$

11. (a)

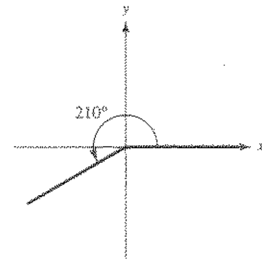


(b) Quadrant I

(c) $45° + 360° = 405°$

$45° - 360° = -315°$

12. (a)

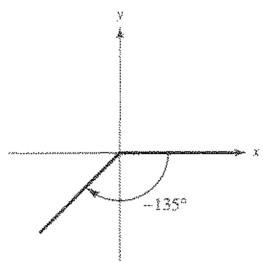


(b) Quadrant III

(c) $210° + 360° = 570°$

$210° - 360° = -150°$

13. (a)

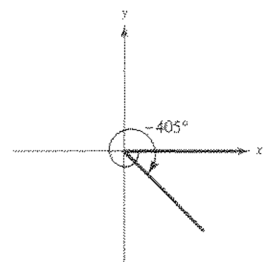


(b) Quadrant III

(c) $-135° + 360° = 225°$

$-135° - 360° = -495°$

14. (a)



(b) Quadrant IV

(c) $-405° + 720° = 315°$

$-405° + 360° = -45°$

15. Complement of 5°: $90° - 5° = 85°$

Supplement of 5°: $180° - 5° = 175°$

16. Complement of 84°: $90° - 84° = 6°$

Supplement of 84°: $180° - 84° = 96°$

17. Complement: not possible

Supplement: $180° - 171° = 9°$

18. Complement of 136°: not possible

Supplement of 136°: $180° - 136° = 44°$

19. $135°\,16'45'' = \left(135 + \dfrac{16}{60} + \dfrac{45}{3600}\right)^{\circ} \approx 135.279°$

20. $-234°\,40'' = -\left(234° + \dfrac{40°}{3600}\right) \approx -234.011°$

21. $5°\,22'\,53'' = \left(5 + \dfrac{22}{60} + \dfrac{53}{3600}\right)^{\circ} \approx 5.381°$

22. $280°\,8'\,50'' = 280° + \dfrac{8°}{60} + \dfrac{50°}{3600}$

$\approx 280° + 0.13° + 0.01° = 280.147°$

23. $135.29° = 135° + (0.29)(60)' = 135°\,17'\,24''$

24. $25.8° = 25°\,48'$

25. $-85.36° = -[85 + 0.36(60')] = -85°\,21'\,36''$

26. $-327.93° = -327°\,55'\,48''$

27. $415° = 415° \cdot \dfrac{\pi \text{ rad}}{180°} = \dfrac{83\pi}{36} \text{ rad} \approx 7.243 \text{ rad}$

28. $-355° = -355° \cdot \dfrac{\pi \text{ rad}}{180°}$

$= -\dfrac{71\pi}{36} \text{ rad} \approx -6.196 \text{ rad}$

29. $-72° = -72° \cdot \dfrac{\pi \text{ rad}}{180°} = -\dfrac{2\pi}{5} \text{ rad} \approx -1.257 \text{ rad}$

30. $94° = 94° \cdot \dfrac{\pi \text{ rad}}{180°} = \dfrac{47\pi}{90} \text{ rad} \approx 1.641 \text{ rad}$

31. $\dfrac{5\pi}{7} = \dfrac{5\pi}{7}\left(\dfrac{180°}{\pi}\right) \approx 128.571°$

32. $-\dfrac{3\pi}{5} = -\dfrac{3\pi}{5}\left(\dfrac{180°}{\pi}\right) = -108°$

33. $-3.5 = -3.5\left(\dfrac{180°}{\pi}\right) \approx -200.535°$

34. $1.55 = 1.55\left(\dfrac{180°}{\pi}\right) \approx 88.808°$

35. $s = r\theta$

$25 = 12\theta$

$\theta = \dfrac{25}{12} \approx 2.083$

36. $s = r\theta \implies \theta = \dfrac{s}{r} = \dfrac{245}{60} = 4.08\overline{3} = \dfrac{49}{12}$ radians

37. $s = r\theta$

$s = 20(138°)\dfrac{\pi}{180°}$

$s \approx 48.171$ m

38. $s = r\theta = (15)\dfrac{\pi}{3} = 5\pi \approx 15.71$ cm

39. In one revolution, the arc length traveled is $s = 2\pi r = 2\pi(6) = 12\pi$ cm. The time required for one revolution is

$t = \dfrac{1}{500}$ minutes $= \dfrac{1}{500}(60) = \dfrac{3}{25}$ seconds.

Linear speed $= \dfrac{s}{t} = \dfrac{12\pi}{3/25} = 100\pi$ cm/sec

40. (a) 28 miles per hour $= \dfrac{28(5280)}{60} = 2464$ ft/min

Circumference of wheel: $C = \pi\left(2\dfrac{1}{3}\right) = \dfrac{7}{3}\pi$

Number of revolutions per minute:

$\dfrac{2464}{(7/3)\pi} = \dfrac{7392}{7\pi} \approx 336.1$ rev/min

(b) $\dfrac{\theta}{t} = \dfrac{7392}{7\pi}(2\pi) = \dfrac{14{,}724}{7} \approx 2112$ rad/min

41. $t = \dfrac{7\pi}{4}$ corresponds to $\left(\dfrac{\sqrt{2}}{2}, -\dfrac{\sqrt{2}}{2}\right)$.

42. $\cos\dfrac{3\pi}{4} = -\dfrac{\sqrt{2}}{2}$, $\sin\dfrac{3\pi}{4} = \dfrac{\sqrt{2}}{2}$

$(x, y) = \left(-\dfrac{\sqrt{2}}{2}, \dfrac{\sqrt{2}}{2}\right)$

43. $\cos\dfrac{5\pi}{6} = -\dfrac{\sqrt{3}}{2}$

$\sin\dfrac{5\pi}{6} = \dfrac{1}{2}$

$(x, y) = \left(-\dfrac{\sqrt{3}}{2}, \dfrac{1}{2}\right)$

44. $t = \dfrac{4\pi}{3}$ corresponds to $\left(-\dfrac{1}{2}, -\dfrac{\sqrt{3}}{2}\right)$.

45. $t = -\dfrac{2\pi}{3}$ corresponds to $\left(-\dfrac{1}{2}, -\dfrac{\sqrt{3}}{2}\right)$.

46. $t = -\dfrac{7\pi}{6}$ corresponds to $\left(-\dfrac{\sqrt{3}}{2}, \dfrac{1}{2}\right)$.

47. $t = -\frac{5\pi}{4}$ corresponds to $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$.

48. $t = -\frac{5\pi}{6}$ corresponds to $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$.

49. $\sin\frac{7\pi}{6} = -\frac{1}{2}$ $\qquad \cot\frac{7\pi}{6} = \sqrt{3}$

$\cos\frac{7\pi}{6} = -\frac{\sqrt{3}}{2}$ $\qquad \sec\frac{7\pi}{6} = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$

$\tan\frac{7\pi}{6} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ $\qquad \csc\frac{7\pi}{6} = -2$

50. $\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} = \cos\frac{\pi}{4}$

$\tan\frac{\pi}{4} = 1 = \cot\frac{\pi}{4}$

$\sec\frac{\pi}{4} = \sqrt{2} = \csc\frac{\pi}{4}$

51. $t = 2\pi$ corresponds to $(1, 0)$.

$\sin 2\pi = y = 0$ $\qquad \cot 2\pi = \frac{x}{y}$, undefined

$\cos 2\pi = x = 1$ $\qquad \sec 2\pi = \frac{1}{x} = 1$

$\tan 2\pi = \frac{y}{x} = 0$ $\qquad \csc 2\pi = \frac{1}{y}$, undefined

52. $t = -\pi$ corresponds to $(-1, 0)$.

$\sin(-\pi) = y = 0$ $\qquad \cot(-\pi) = \frac{x}{y}$, undefined

$\cos(-\pi) = x = -1$ $\qquad \sec(-\pi) = \frac{1}{x} = -1$

$\tan(-\pi) = \frac{y}{x} = 0$ $\qquad \csc(-\pi) = \frac{1}{y}$, undefined

53. $t = -\frac{11\pi}{6}$ corresponds to $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$.

$\sin\left(-\frac{11\pi}{6}\right) = y = \frac{1}{2}$

$\cos\left(-\frac{11\pi}{6}\right) = x = \frac{\sqrt{3}}{2}$

$\tan\left(-\frac{11\pi}{6}\right) = \frac{y}{x} = \frac{\sqrt{3}}{3}$

$\cot\left(-\frac{11\pi}{6}\right) = \frac{x}{y} = \sqrt{3}$

$\sec\left(-\frac{11\pi}{6}\right) = \frac{1}{x} = \frac{2\sqrt{3}}{3}$

$\csc\left(-\frac{11\pi}{6}\right) = \frac{1}{y} = 2$

54. $t = -\frac{5\pi}{6}$ corresponds to $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$.

$\sin\left(-\frac{5\pi}{6}\right) = y = -\frac{1}{2}$

$\cos\left(-\frac{5\pi}{6}\right) = x = -\frac{\sqrt{3}}{2}$

$\tan\left(-\frac{5\pi}{6}\right) = \frac{y}{x} = \frac{\sqrt{3}}{3}$

$\cot\left(-\frac{5\pi}{6}\right) = \frac{x}{y} = \sqrt{3}$

$\sec\left(-\frac{5\pi}{6}\right) = \frac{1}{x} = -\frac{2\sqrt{3}}{3}$

$\csc\left(-\frac{5\pi}{6}\right) = \frac{1}{y} = -2$

55. $t = -\frac{\pi}{2}$ corresponds to $(0, -1)$.

$\sin\left(-\frac{\pi}{2}\right) = y = -1$ $\qquad \cot\left(-\frac{\pi}{2}\right) = \frac{x}{y} = 0$

$\cos\left(-\frac{\pi}{2}\right) = x = 0$ $\qquad \sec\left(-\frac{\pi}{2}\right) = \frac{1}{x}$, undefined

$\tan\left(-\frac{\pi}{2}\right) = \frac{y}{x}$, undefined $\qquad \csc\left(-\frac{\pi}{2}\right) = \frac{1}{y} = -1$

56. $t = -\frac{\pi}{4}$ corresponds to $\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$.

$\sin\left(-\frac{\pi}{4}\right) = y = -\frac{\sqrt{2}}{2}$ $\qquad$ $\cot\left(-\frac{\pi}{4}\right) = \frac{x}{y} = -1$

$\cos\left(-\frac{\pi}{4}\right) = x = \frac{\sqrt{2}}{2}$ $\qquad$ $\sec\left(-\frac{\pi}{4}\right) = \frac{1}{x} = \sqrt{2}$

$\tan\left(-\frac{\pi}{4}\right) = \frac{y}{x} = -1$ $\qquad$ $\csc\left(-\frac{\pi}{4}\right) = \frac{1}{y} = -\sqrt{2}$

57. $\sin\left(\frac{11\pi}{4}\right) = \sin\left(\frac{3\pi}{4}\right) = \frac{\sqrt{2}}{2}$

58. $\cos 4\pi = \cos 0 = 1$

59. $\sin\left(-\frac{17\pi}{6}\right) = \sin\left(\frac{7\pi}{6}\right) = -\frac{1}{2}$

60. $\cos\left(-\frac{13\pi}{3}\right) = \cos\frac{5\pi}{3} = \frac{1}{2}$

61. $\sin t = \frac{3}{5}$

(a) $\sin(-t) = -\sin t = -\frac{3}{5}$

(b) $\csc(-t) = \frac{1}{\sin(-t)} = -\frac{5}{3}$

62. $\cos t = \frac{5}{13}$

(a) $\cos(-t) = \cos t = \frac{5}{13}$

(b) $\sec(-t) = \frac{1}{\cos(-t)} = \frac{13}{5}$

63. $\sin(-t) = -\frac{2}{3}$

(a) $\sin t = -\sin(-t) = -\left(-\frac{2}{3}\right) = \frac{2}{3}$

(b) $\csc t = \frac{1}{\sin t} = \frac{3}{2}$

64. $\cos(-t) = \frac{5}{8}$

(a) $\cos t = \cos(-t) = \frac{5}{8}$

(b) $\sec(-t) = \frac{1}{\cos(-t)} = \frac{8}{5}$

65. $\cot 2.3 = \frac{1}{\tan 2.3} \approx -0.8935$

66. $\sec 4.5 = \frac{1}{\cos 4.5} \approx -4.7439$

67. $\cos\frac{5\pi}{3} = \frac{1}{2}$

68. $\tan\left(-\frac{11\pi}{6}\right) \approx 0.5774$

69. The opposite side is $\sqrt{9^2 - 4^2} = \sqrt{81 - 16} = \sqrt{65}$.

$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{\sqrt{65}}{9}$ $\qquad$ $\csc\theta = \frac{1}{\sin\theta} = \frac{9}{\sqrt{65}} = \frac{9\sqrt{65}}{65}$

$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{4}{9}$ $\qquad$ $\sec\theta = \frac{1}{\cos\theta} = \frac{9}{4}$

$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{\sqrt{65}}{4}$ $\qquad$ $\cot\theta = \frac{1}{\tan\theta} = \frac{4}{\sqrt{65}} = \frac{4\sqrt{65}}{65}$

70. $\sin\theta = \dfrac{15}{3\sqrt{41}} = \dfrac{5\sqrt{41}}{41}$

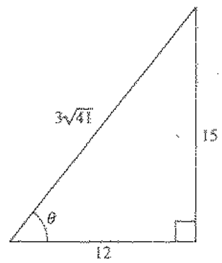
$\cos\theta = \dfrac{12}{3\sqrt{41}} = \dfrac{4\sqrt{41}}{41}$

$\tan\theta = \dfrac{15}{12} = \dfrac{5}{4}$

$\csc\theta = \dfrac{41}{5\sqrt{41}} = \dfrac{\sqrt{41}}{5}$

$\sec\theta = \dfrac{41}{4\sqrt{41}} = \dfrac{\sqrt{41}}{4}$

$\cot\theta = \dfrac{4}{5}$



71. The hypotenuse is $\sqrt{12^2 + 10^2} = \sqrt{244} = 2\sqrt{61}$.

$\sin\theta = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{10}{2\sqrt{61}} = \dfrac{5}{\sqrt{61}} = \dfrac{5\sqrt{61}}{61}$

$\cos\theta = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{12}{2\sqrt{61}} = \dfrac{6}{\sqrt{61}} = \dfrac{6\sqrt{61}}{61}$

$\tan\theta = \dfrac{\text{opp}}{\text{adj}} = \dfrac{10}{12} = \dfrac{5}{6}$

$\csc\theta = \dfrac{1}{\sin\theta} = \dfrac{\sqrt{61}}{5}$

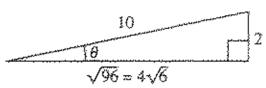
$\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{\sqrt{61}}{6}$

$\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{6}{5}$

72. $\sin\theta = \dfrac{2}{10} = \dfrac{1}{5}$ $\qquad \csc\theta = 5$

$\cos\theta = \dfrac{4\sqrt{6}}{10} = \dfrac{2\sqrt{6}}{5}$ $\qquad \sec\theta = \dfrac{5}{2\sqrt{6}} = \dfrac{5\sqrt{6}}{12}$

$\tan\theta = \dfrac{2}{4\sqrt{6}} = \dfrac{\sqrt{6}}{12}$ $\qquad \cot\theta = \dfrac{12}{\sqrt{6}} = 2\sqrt{6}$



73. $\csc\theta\tan\theta = \dfrac{1}{\sin\theta}\cdot\dfrac{\sin\theta}{\cos\theta} = \dfrac{1}{\cos\theta} = \sec\theta$

74. $\dfrac{\cot\theta + \tan\theta}{\cot\theta} = 1 + \dfrac{\tan\theta}{\cot\theta} = 1 + \tan^2\theta = \sec^2\theta$

75. (a) $\cos 84^\circ \approx 0.1045$

(b) $\sin 6^\circ \approx 0.1045$

76. (a) $\csc(52^\circ\, 12') = \dfrac{1}{\sin(52^\circ\, 12')} = \dfrac{1}{\sin 52.2^\circ} \approx 1.2656$

(b) $\sec(54^\circ\, 7') = \dfrac{1}{\cos(54^\circ\, 7')} = \dfrac{1}{\cos(54.1167^\circ)} \approx 1.7061$

77. (a) $\cos\dfrac{\pi}{4} \approx 0.7071$

(b) $\sec\dfrac{\pi}{4} \approx 1.4142$

78. (a) $\tan\left(\dfrac{3\pi}{20}\right) \approx 0.5095$

(b) $\cot\left(\dfrac{3\pi}{20}\right) = \dfrac{1}{\tan(3\pi/20)}$

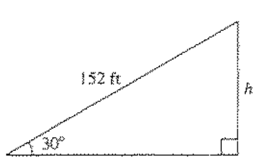
≈ 1.9626

79. $\tan 62^\circ = \dfrac{\omega}{125}$

$x = 125\tan 62^\circ$

≈ 235 feet

80. (a)



(b) $\sin 30^\circ = \dfrac{h}{152} \Longrightarrow h = 152\sin 30^\circ$

(c) $h = 152\left(\dfrac{1}{2}\right) = 76$ feet

81. $x = 12, y = 16, r = \sqrt{144 + 256} = \sqrt{400} = 20$

$\sin \theta = \frac{y}{r} = \frac{4}{5}$ $\qquad \csc \theta = \frac{r}{y} = \frac{5}{4}$

$\cos \theta = \frac{x}{r} = \frac{3}{5}$ $\qquad \sec \theta = \frac{r}{x} = \frac{5}{3}$

$\tan \theta = \frac{y}{x} = \frac{4}{3}$ $\qquad \cot \theta = \frac{x}{y} = \frac{3}{4}$

82. $x = 2, y = 10, r = \sqrt{4 + 100} = \sqrt{104} = 2\sqrt{26}$

$\sin \theta = \frac{y}{r} = \frac{10}{2\sqrt{26}} = \frac{5\sqrt{26}}{26}$

$\cos \theta = \frac{x}{r} = \frac{2}{2\sqrt{26}} = \frac{\sqrt{26}}{26}$

$\tan \theta = \frac{y}{x} = \frac{10}{2} = 5$

$\csc \theta = \frac{r}{y} = \frac{\sqrt{26}}{5}$

$\sec \theta = \frac{r}{x} = \sqrt{26}$

$\cot \theta = \frac{x}{y} = \frac{1}{5}$

83. $x = -7, \; y = 2, \; r = \sqrt{49 + 4} = \sqrt{53}$

$\sin \theta = \frac{y}{r} = \frac{2}{\sqrt{53}} = \frac{2\sqrt{53}}{53}$

$\cos \theta = \frac{x}{r} = -\frac{7}{\sqrt{53}} = -\frac{7\sqrt{53}}{53}$

$\tan \theta = \frac{y}{x} = -\frac{2}{7}$

$\csc \theta = \frac{\sqrt{53}}{2}$

$\sec \theta = -\frac{\sqrt{53}}{7}$

$\cot \theta = -\frac{7}{2}$

84. $x = 3, y = -4, r = \sqrt{3^2 + (-4)^2} = 5$

$\sin \theta = \frac{y}{r} = -\frac{4}{5}$

$\cos \theta = \frac{x}{r} = \frac{3}{5}$

$\tan \theta = \frac{y}{x} = -\frac{4}{3}$

$\csc \theta = \frac{x}{y} = -\frac{5}{4}$

$\sec \theta = \frac{r}{x} = \frac{5}{3}$

$\cot \theta = \frac{r}{y} = -\frac{3}{4}$

85. $x = \frac{2}{3}$ and $y = \frac{5}{8}, \; r = \sqrt{\left(\frac{2}{3}\right)^2 + \left(\frac{5}{8}\right)^2} = \frac{\sqrt{481}}{24}$

$\sin \theta = \frac{y}{r} = \frac{5/8}{\sqrt{481}/24} = \frac{15\sqrt{481}}{481}$

$\cos \theta = \frac{x}{r} = \frac{2/3}{\sqrt{481}/24} = \frac{16\sqrt{481}}{481}$

$\tan \theta = \frac{y}{x} = \frac{5/8}{2/3} = \frac{15}{16}$

$\csc \theta = \frac{r}{y} = \frac{\sqrt{481}}{15}$

$\sec \theta = \frac{r}{x} = \frac{\sqrt{481}}{16}$

$\cot \theta = \frac{x}{y} = \frac{16}{15}$

86. $x = -\frac{10}{3}, y = -\frac{2}{3}, r = \sqrt{\left(-\frac{10}{3}\right)^2 + \left(-\frac{2}{3}\right)^2} = \frac{2\sqrt{26}}{3}$

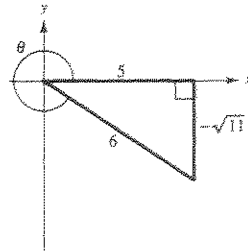
$\sin\theta = \frac{y}{r} = \frac{-2/3}{2\sqrt{26}/3} = \frac{-1}{\sqrt{26}} = \frac{-\sqrt{26}}{26}$ $\qquad \csc\theta = \frac{r}{y} = -\sqrt{26}$

$\cos\theta = \frac{x}{r} = \frac{-10/3}{2\sqrt{26}/3} = \frac{-5}{\sqrt{26}} = \frac{-5\sqrt{26}}{26}$ $\qquad \sec\theta = \frac{r}{x} = -\frac{\sqrt{26}}{5}$

$\tan\theta = \frac{y}{x} = \frac{-2/3}{-10/3} = \frac{1}{5}$ $\qquad \cot\theta = \frac{x}{y} = 5$

87. $\sec\theta = \frac{6}{5}$, $\tan\theta < 0 \implies \theta$ is in Quadrant IV.

$r = 6, x = 5, y = -\sqrt{36 - 25} = -\sqrt{11}$



$\sin\theta = \frac{y}{r} = -\frac{\sqrt{11}}{6}$ $\qquad \csc\theta = -\frac{6\sqrt{11}}{11}$

$\cos\theta = \frac{x}{r} = \frac{5}{6}$ $\qquad \sec\theta = \frac{6}{5}$

$\tan\theta = \frac{y}{x} = -\frac{\sqrt{11}}{5}$ $\qquad \cot\theta = -\frac{5\sqrt{11}}{11}$

88. $\tan\theta = \frac{y}{x} = -\frac{12}{5} \implies r = 13, \sin\theta > 0 \implies y = 12, x = -5$

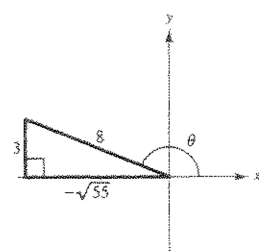
$\sin\theta = \frac{y}{r} = \frac{12}{13}$ $\qquad \csc\theta = \frac{r}{y} = \frac{13}{12}$

$\cos\theta = \frac{x}{r} = -\frac{5}{13}$ $\qquad \sec\theta = \frac{r}{x} = \frac{13}{-5} = -\frac{13}{5}$

$\qquad \cot\theta = \frac{x}{y} = -\frac{5}{12}$

89. $\sin\theta = \frac{3}{8}$, $\cos\theta < 0 \implies \theta$ is in Quadrant II.

$y = 3, r = 8, x = -\sqrt{55}$



$\sin\theta = \frac{y}{r} = \frac{3}{8}$ $\qquad \csc\theta = \frac{8}{3}$

$\cos\theta = \frac{x}{r} = -\frac{\sqrt{55}}{8}$ $\qquad \sec\theta = -\frac{8}{\sqrt{55}} = -\frac{8\sqrt{55}}{55}$

$\tan\theta = \frac{y}{x} = -\frac{3}{\sqrt{55}} = -\frac{3\sqrt{55}}{55}$ $\qquad \cot\theta = -\frac{\sqrt{55}}{3}$

90. $\cos\theta = \dfrac{x}{r} = \dfrac{-2}{5} \implies y = \pm\sqrt{21},\ \sin\theta > 0 \implies y = \sqrt{21}$

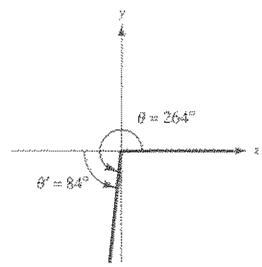
$\sin\theta = \dfrac{y}{r} = \dfrac{\sqrt{21}}{5}$ $\qquad$ $\sec\theta = \dfrac{r}{x} = \dfrac{5}{-2} = -\dfrac{5}{2}$

$\tan\theta = \dfrac{y}{x} = -\dfrac{\sqrt{21}}{2}$ $\qquad$ $\cot\theta = \dfrac{x}{y} = \dfrac{-2}{\sqrt{21}} = -\dfrac{2\sqrt{21}}{21}$

$\csc\theta = \dfrac{r}{y} = \dfrac{5}{\sqrt{21}} = \dfrac{5\sqrt{21}}{21}$

91. Reference angle:

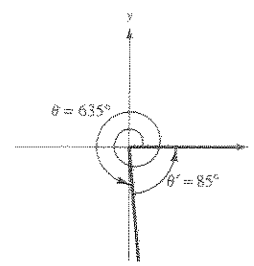
$264° - 180° = 84°$



92. 635° is coterminal with 275°.

Reference angle:

$360° - 275° = 85°$

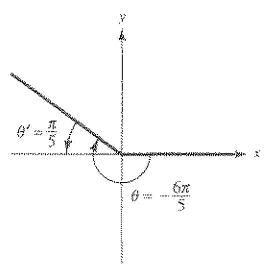


93. Coterminal angle:

$2\pi - \dfrac{6\pi}{5} = \dfrac{4\pi}{5}$

Reference angle:

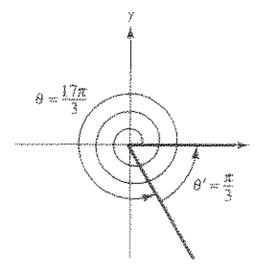
$\pi - \dfrac{4\pi}{5} = \dfrac{\pi}{5}$



94. $\dfrac{17\pi}{3}$ is coterminal with $\dfrac{5\pi}{3}$.

Reference angle:

$2\pi - \dfrac{5\pi}{3} = \dfrac{\pi}{3}$



95. 240° is in Quadrant III with reference angle 60°.

$\sin 240° = -\sin 60° = -\dfrac{\sqrt{3}}{2}$

$\cos 240° = -\cos 60° = -\dfrac{1}{2}$

$\tan 240° = \dfrac{-\sqrt{3}/2}{-1/2} = \sqrt{3}$

96. 315° is in Quadrant IV with reference angle 45°.

$\sin 315° = -\sin 45° = -\dfrac{\sqrt{2}}{2}$

$\cos 315° = \cos 45° = \dfrac{\sqrt{2}}{2}$

$\tan 315° = -\tan 45° = -1$

97. $-210°$ is coterminal with 150° in Quadrant II with reference angle 30°.

$\sin(-210°) = \sin(30°) = \dfrac{1}{2}$

$\cos(-210°) = -\cos(30°) = -\dfrac{\sqrt{3}}{2}$

$\tan(-210°) = \dfrac{1/2}{-\sqrt{3}/2} = -\dfrac{1}{\sqrt{3}} = -\dfrac{\sqrt{3}}{3}$

98. $-315°$ is coterminal with 45° in Quadrant I.

$\sin(-315°) = \dfrac{\sqrt{2}}{2}$

$\cos(-315°) = \dfrac{\sqrt{2}}{2}$

$\tan(-315°) = 1$

99. $-9\pi/4$ is coterminal with $7\pi/4$ in Quadrant IV with reference angle $\pi/4$.

$$\sin\left(-\frac{9\pi}{4}\right) = -\sin\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$$

$$\cos\left(-\frac{9\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

$$\tan\left(-\frac{9\pi}{4}\right) = \frac{-\sqrt{2}/2}{\sqrt{2}/2} = -1$$

100. $11\pi/6$ is in Quadrant IV.

$$\sin\left(\frac{11\pi}{6}\right) = -\frac{1}{2}$$

$$\cos\left(\frac{11\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

$$\tan\left(\frac{11\pi}{6}\right) = -\frac{\sqrt{3}}{3}$$

101. $\sin(4\pi) = \sin(0) = 0$

$\cos(4\pi) = 1$

$\tan(4\pi) = 0$

102. $\sin\left(\frac{7\pi}{3}\right) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

$\cos\left(\frac{7\pi}{3}\right) = \frac{1}{2}$

$\tan\left(\frac{7\pi}{3}\right) = \sqrt{3}$

103. $\tan 33° \approx 0.6494$

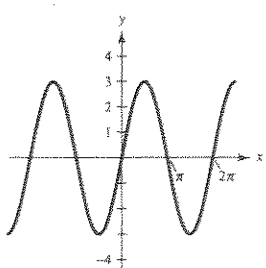
104. $\csc 105° = \dfrac{1}{\sin 105°} \approx 1.0353$

105. $\sec \dfrac{12\pi}{5} = \dfrac{1}{\cos(12\pi/5)} \approx 3.2361$

106. $\sin\left(-\frac{\pi}{9}\right) \approx -0.3420$

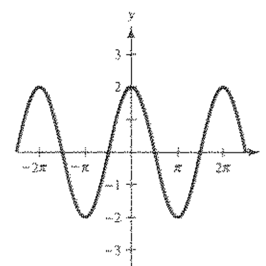
107. $f(x) = 3 \sin x$

Amplitude: 3



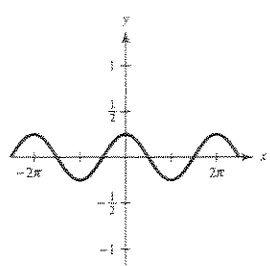
108. $f(x) = 2 \cos x$

Amplitude: 2



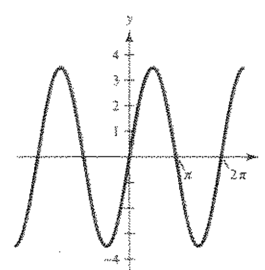
109. $f(x) = \frac{1}{4} \cos x$

Amplitude: $\frac{1}{4}$



110. $f(x) = \frac{7}{2} \sin x$

Amplitude: $\frac{7}{2}$



111. Period: $\dfrac{2\pi}{\pi} = 2$

Amplitude: 5

112. Period: $\dfrac{2\pi}{(1/2)} = 4\pi$

Amplitude: $\dfrac{3}{2}$

113. Period: $\dfrac{2\pi}{2} = \pi$

Amplitude: 3.4

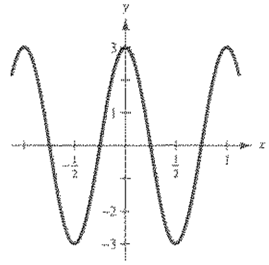
114. Period: $\dfrac{2\pi}{(\pi/2)} = 4$

Amplitude: 4

115. $y = 3\cos 2\pi x$

Amplitude: 3

Period: $\dfrac{2\pi}{2\pi} = 1$



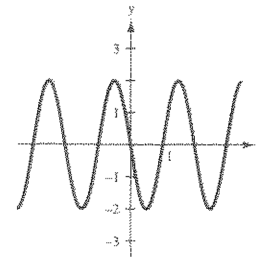
116. $y = -2\sin \pi x$

Period: $\dfrac{2\pi}{\pi} = 2$

Amplitude: $|-2| = 2$

Reflected in x-axis

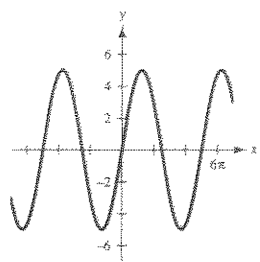
x	$-\frac{1}{2}$	0	$\frac{1}{2}$
y	2	0	-2



117. $f(x) = 5\sin\dfrac{2x}{5}$

Amplitude: 5

Period: $\dfrac{2\pi}{2/5} = 5\pi$



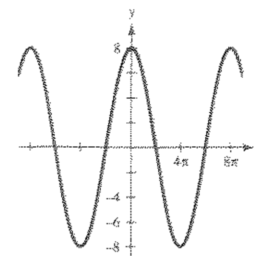
118. $f(x) = 8\cos\left(-\dfrac{x}{4}\right)$

Period: $\dfrac{2\pi}{(1/4)} = 8\pi$

Amplitude: 8

Reflected in y-axis

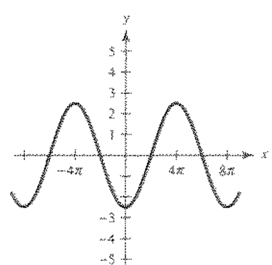
x	-4π	-2π	0	2π	4π
y	-8	0	8	0	-8



119. $f(x) = -\dfrac{5}{2}\cos\left(\dfrac{x}{4}\right)$

Amplitude: $\dfrac{5}{2}$

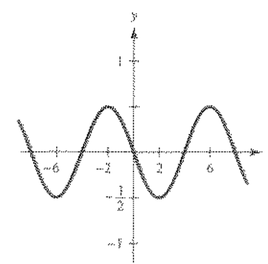
Period: $\dfrac{2\pi}{1/4} = 8\pi$



120. $f(x) = -\dfrac{1}{2}\sin\dfrac{\pi x}{4}$

Amplitude: $\dfrac{1}{2}$

Period: 8



121. $f(x) = \dfrac{5}{2}\sin(x - \pi)$

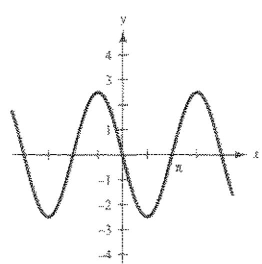
Amplitude: $\dfrac{5}{2}$

Period: 2π

Shift:

$x - \pi = 0$ and $x - \pi = 2\pi$

$x = \pi \qquad\qquad x = 3\pi$



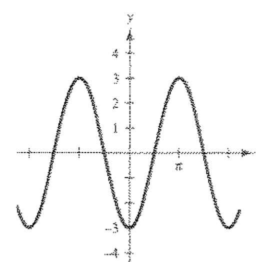
122. $f(x) = 3\cos(x + \pi)$

Period: 2π

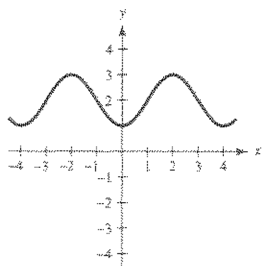
Amplitude: 3

This is the graph of $y = 3\cos x$ shifted to the left π units.

x	$-\pi$	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$	π
$f(x)$	3	0	-3	0	3



123. $f(x) = 2 - \cos \dfrac{\pi x}{2}$

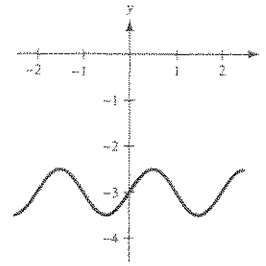


124. $f(x) = \dfrac{1}{2} \sin \pi x - 3$

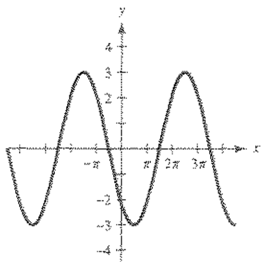
Amplitude: $\dfrac{1}{2}$

Period: 2

Vertical shift downward three units



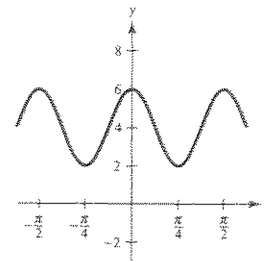
125. $f(x) = -3 \cos\left(\dfrac{x}{2} - \dfrac{\pi}{4}\right)$



126. $f(x) = 4 - 2\cos(4x + \pi)$

Amplitude: 2

Period: $\dfrac{\pi}{2}$



127. $f(x) = -2\cos\left(x - \dfrac{\pi}{4}\right)$

128. $f(x) = a\cos(bx - c)$

Amplitude: 3

Period: $\pi \implies f(x) = 3\cos(2x)$

129. $f(x) = -4\cos\left(2x - \dfrac{\pi}{2}\right)$

130. $f(x) = a\cos(bx - c)$

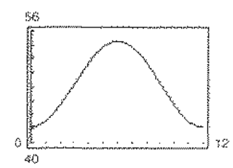
Amplitude: $\dfrac{1}{2}$

Period: $2 \implies f(x) = \dfrac{1}{2}\cos \pi x$

131. $S = 48.4 - 6.1 \cos \dfrac{\pi t}{6}$

Maximum sales: $t = 6$ (June)

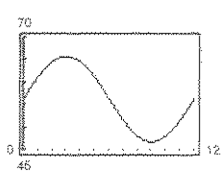
Minimum sales: $t = 12$ (December)



132. $S = 56.25 + 9.50 \sin \dfrac{\pi t}{6}$

Maximum sales: $t = 3$ (March)

Minimum sales: $t = 9$ (September)



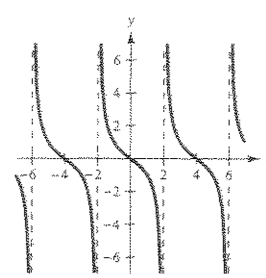
133. $f(x) = -\tan \dfrac{\pi x}{4}$

Period: $\dfrac{\pi}{(\pi/4)} = 4$

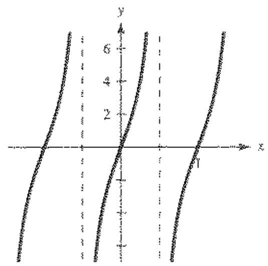
Asymptotes: $x = -2, x = 2$

Reflected in x-axis

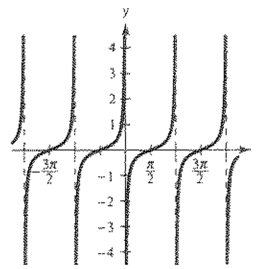
x	-1	0	1
y	1	0	-1



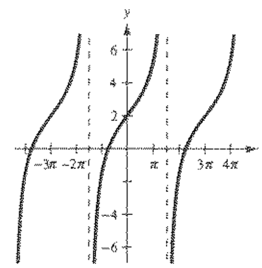
134. $f(x) = 4 \tan \pi x$



135. $f(x) = \frac{1}{4} \tan\left(x - \frac{\pi}{2}\right)$



136. $f(x) = 2 + 2 \tan \frac{x}{3}$



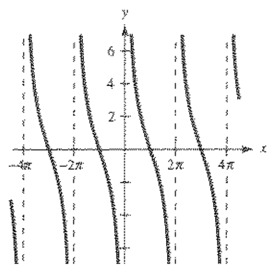
137. $f(x) = 3 \cot \frac{x}{2}$

Period: $\frac{\pi}{1/2} = 2\pi$

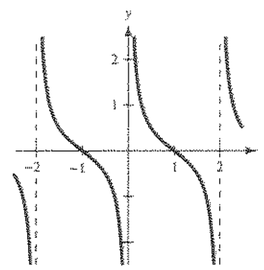
Two consecutive asymptotes:

$\frac{x}{2} = 0 \implies x = 0$

$\frac{x}{2} = \pi \implies x = 2\pi$



138. $f(x) = \frac{1}{2} \cot \frac{\pi x}{2} = \frac{1}{2 \tan \frac{\pi x}{2}}$



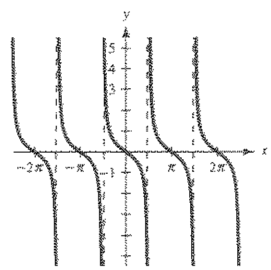
139. $f(x) = \frac{1}{2} \cot\left(x - \frac{\pi}{2}\right)$

Period: π

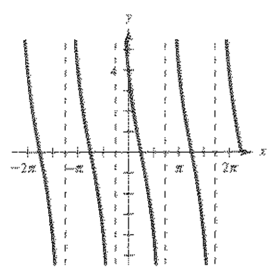
Two consecutive asymptotes:

$x - \frac{\pi}{2} = 0 \implies x = \frac{\pi}{2}$

$x - \frac{\pi}{2} = \pi \implies x = \frac{3\pi}{2}$

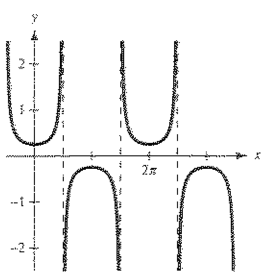


140. $f(x) = 4 \cot\left(x + \frac{\pi}{4}\right) = \frac{4}{\tan\left(x + \frac{\pi}{4}\right)}$

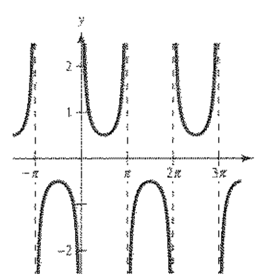


141. $f(x) = \frac{1}{4} \sec x$

Period: 2π

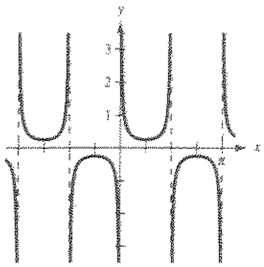


142. $f(x) = \frac{1}{2} \csc x = \frac{1}{2 \sin x}$

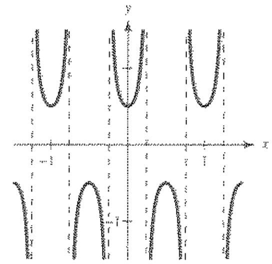


143. $f(x) = \frac{1}{4}\csc 2x$

Period: π

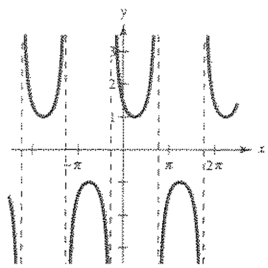


144. $f(x) = \frac{1}{2}\sec 2\pi x = \frac{1}{2\cos 2\pi x}$

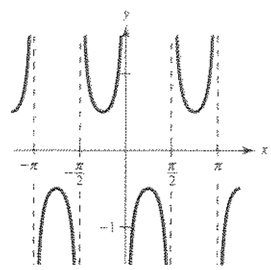


145. $f(x) = \sec\left(x - \frac{\pi}{4}\right)$

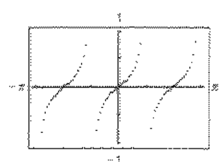
Secant function shifted $\frac{\pi}{4}$ to right



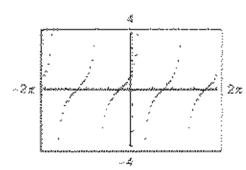
146. $f(x) = \frac{1}{2}\csc(2x + \pi) = \frac{1}{2\sin(2x + \pi)}$



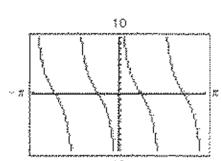
147. $f(x) = \frac{1}{4}\tan\frac{\pi x}{2}$



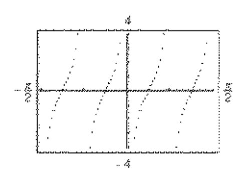
148. $f(x) = \tan\left(x + \frac{\pi}{4}\right)$



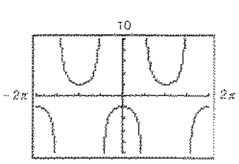
149. $f(x) = 4\cot(2x - \pi) = \frac{4}{\tan(2x - \pi)}$



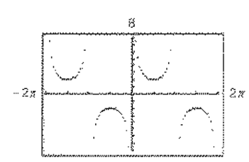
150. $f(x) = -2\cot(4x + \pi) = -\frac{2}{\tan(4x + \pi)}$



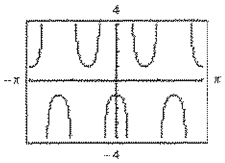
151. $f(x) = 2\sec(x - \pi) = \frac{2}{\cos(x - \pi)}$



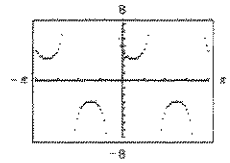
152. $f(x) = -2\csc(x - \pi) = \frac{-2}{\sin(x - \pi)}$



153. $f(x) = \csc\left(3x - \frac{\pi}{2}\right) = \frac{1}{\sin(3x - (\pi/2))}$

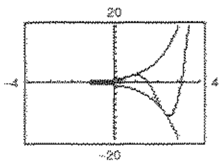


154. $f(x) = 3\csc\left(2x + \frac{\pi}{4}\right) = \frac{3}{\sin(2x + (\pi/4))}$



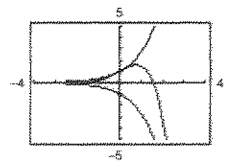
155. $f(x) = e^x \sin 2x$

Damping factor: $y = e^x$



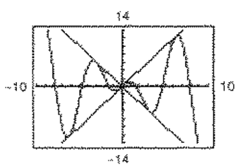
156. $f(x) = e^x \cos x$

Damping factor: e^x



157. $f(x) = 2x \cos x$

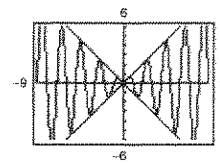
Damping factor: $g(x) = 2x$



As $x \to \infty$, f oscillates between $2x$ and $-2x$.

158. $f(x) = x \sin \pi x$

As $x \to \infty$, f oscillates between $-x$ and x.



159. (a) $\arcsin(-1) = -\frac{\pi}{2}$ because $\sin\left(-\frac{\pi}{2}\right) = -1$.

(b) arcsin 4 does not exist because the domain of arcsin is $[-1, 1]$.

160. (a) $\arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$ because $\sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$.

(b) $\arcsin\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$ because

$\sin\left(-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$.

161. (a) $\arccos\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$ because $\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$.

(b) $\arccos\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$ because $\cos\frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$.

162. (a) $\arctan(-\sqrt{3}) = -\frac{\pi}{3}$

(b) $\arctan(1) = \frac{\pi}{4}$

163. $\arccos(0.42) \approx 1.14$

164. $\arcsin 0.63 \approx 0.68$

165. $\sin^{-1}(-0.94) \approx -1.22$

166. $\cos^{-1}(-0.12) \approx 1.69$

167. $\arctan(-12) \approx -1.49$

168. $\arctan 21 \approx 1.52$

169. $\tan^{-1}(0.81) \approx 0.68$

170. $\tan^{-1} 6.4 \approx 1.42$

171. $\sin\theta = \frac{x+3}{16} \implies \theta = \arcsin\left(\frac{x+3}{16}\right)$

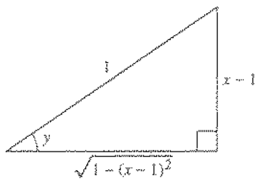
172. $\tan\theta = \frac{x+1}{20} \implies \theta = \arctan\left(\frac{x+1}{20}\right)$

173. Let $y = \arcsin(x - 1)$. Then,

$$\sin y = (x - 1) = \frac{x - 1}{1} \text{ and}$$

$$\sec y = \frac{1}{\sqrt{-x^2 + 2x}}$$

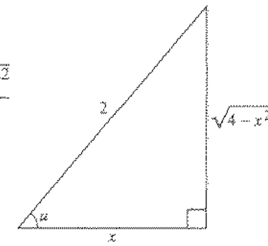
$$= \frac{\sqrt{-x^2 + 2x}}{-x^2 + 2x}.$$



174. Let $u = \arccos \frac{x}{2}$, $\cos u = \frac{x}{2}$.

$$\tan\left(\arccos \frac{x}{2}\right) = \tan u$$

$$= \frac{\sqrt{4 - x^2}}{x}$$

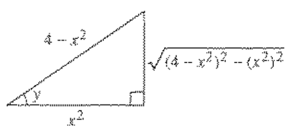


175. Let $y = \arccos \frac{x^2}{4 - x^2}$. Then $\cos y = \frac{x^2}{4 - x^2}$ and

$$\sin y = \frac{\sqrt{(4 - x^2)^2 - (x^2)^2}}{4 - x^2}.$$

$$= \frac{\sqrt{16 - 8x^2}}{4 - x^2}$$

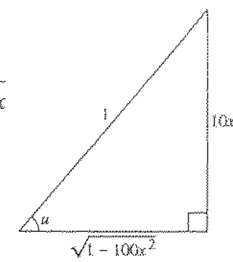
$$= \frac{2\sqrt{4 - 2x^2}}{4 - x^2}.$$



176. Let $u = \arcsin 10x$, $\sin u = 10x$.

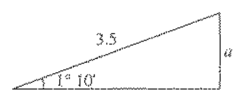
$$\csc(\arcsin 10x) = \csc u$$

$$= \frac{\text{hyp}}{\text{opp}} = \frac{1}{10x}$$



177. $\sin(1° \, 10') = \frac{a}{3.5}$

$$a = 3.5 \sin(1° \, 10') = 3.5 \sin\left(\frac{7°}{6}\right) \approx 0.0713 \text{ or } 71 \text{ meters}$$

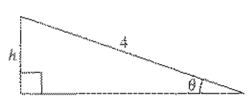


not drawn to scale

178. $\tan \theta = \frac{12}{100}$

$$\theta = \arctan\left(\frac{12}{100}\right) \approx 0.1194 \text{ or } 6.84°$$

$$\sin(\theta) = \frac{h}{4}$$

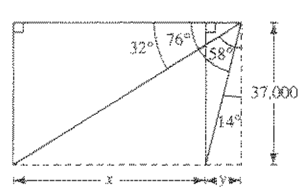


$h = 4 \sin(0.1194) \approx 0.48$ miles or 2534 feet (answer depends on angle θ accuracy)

179. $\tan 14° = \frac{y}{37{,}000} \implies y = 37{,}000 \tan 14° \approx 9225.1$ feet

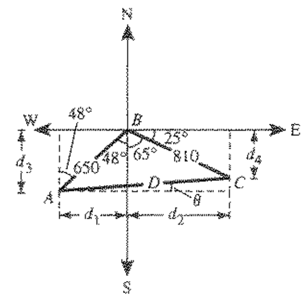
$$\tan 58° = \frac{x + y}{37{,}000} \implies x + y = 37{,}000 \tan 58° \approx 59{,}212.4 \text{ feet}$$

$$x = 59{,}212.4 - 9225.1 \approx 49{,}987.2 \text{ feet}$$



The towns are approximately 50,000 feet apart or 9.47 miles.

180. $\left.\begin{array}{l}\sin 48^\circ = \dfrac{d_1}{650} \implies d_1 \approx 483 \\ \cos 25^\circ = \dfrac{d_2}{810} \implies d_2 \approx 734\end{array}\right\} d_1 + d_2 = 1217$



$\left.\begin{array}{l}\cos 48^\circ = \dfrac{d_3}{650} \implies d_3 \approx 435 \\ \sin 25^\circ = \dfrac{d_4}{810} \implies d_4 \approx 342\end{array}\right\} d_3 - d_4 \approx 93$

$\tan \theta \approx \dfrac{93}{1217} \implies \theta \approx 4.4^\circ$

$\sec 4.4^\circ \approx \dfrac{D}{1217} \implies D \approx 1217 \sec 4.4^\circ \approx 1221$

The distance is 1221 miles and the bearing is N 85.6 E.

181. Use cosine model with amplitude 3 feet.

Period: 15 seconds

$y = 3\cos\left(\dfrac{2\pi}{15}t\right)$

182. Use a cosine model with amplitude $\dfrac{1.5}{2} = 0.75$.

Period: 3 seconds

$y = 0.75\cos\left(\dfrac{2\pi}{3}t\right)$

183. False. $y = \sin\theta$ is a function, but it is not one-to-one.

184. False. The sine and cosine functions are useful for modeling simple harmonic motion.

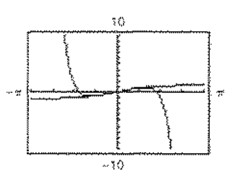
185. $\tan\theta = \dfrac{0.672s^2}{3000}$

(a)

s	10	20	30	40	50	60
θ	1.28°	5.12°	11.40°	19.72°	29.25°	38.88°

(b) θ increases at an increasing rate. The function is not linear.

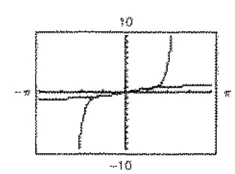
186. (a)



(b) Next term: $\dfrac{x^9}{9}$

$\arctan x \approx x - \dfrac{x^3}{3} + \dfrac{x^5}{5} - \dfrac{x^7}{7} + \dfrac{x^9}{9}$

The accuracy of the approximation increases as more terms are added.



Chapter 4 Practice Test

1. Express 350° in radian measure.

2. Express $(5\pi)/9$ in degree measure.

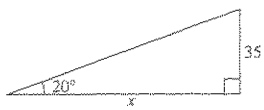
3. Convert 135° 14′ 12″ to decimal form.

4. Convert $-22.569°$ to D° M′ S″ form.

5. If $\cos \theta = \frac{2}{3}$, use the trigonometric identities to find $\tan \theta$.

6. Find θ given $\sin \theta = 0.9063$.

7. Solve for x in the figure below.



8. Find the magnitude of the reference angle for $\theta = (6\pi)/5$.

9. Evaluate csc 3.92.

10. Find $\sec \theta$ given that θ lies in Quadrant III and $\tan \theta = 6$.

11. Graph $y = 3 \sin \dfrac{x}{2}$.

12. Graph $y = -2 \cos(x - \pi)$.

13. Graph $y = \tan 2x$.

14. Graph $y = -\csc\left(x + \dfrac{\pi}{4}\right)$.

15. Graph $y = 2x + \sin x$, using a graphing calculator.

16. Graph $y = 3x \cos x$, using a graphing calculator.

17. Evaluate arcsin 1.

18. Evaluate $\arctan(-3)$.

19. Evaluate $\sin\left(\arccos \dfrac{4}{\sqrt{35}}\right)$.

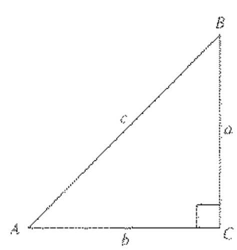
20. Write an algebraic expression for $\cos\left(\arcsin \dfrac{x}{4}\right)$.

For Exercises 21–23, solve the right triangle.

21. $A = 40°$, $c = 12$

22. $B = 6.84°$, $a = 21.3$

23. $a = 5$, $b = 9$



24. A 20-foot ladder leans against the side of a barn. Find the height of the top of the ladder if the angle of elevation of the ladder is 67°.

25. An observer in a lighthouse 250 feet above sea level spots a ship off the shore. If the angle of depression to the ship is 5°, how far out is the ship?

CHAPTER 5
Analytic Trigonometry

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